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XV. *On Electromagnetic Induction in Plane, Cylindrical, and Spherical Current-Sheets, and its Representation by Moving Trails of Images.*—Parts II. & III. *By* G. H. Bryan, *Sc.D., F.R.S.**

Part II. Cylindrical Sheets.

In the Proceedings of the Physical Society for 1894 (vol. xiii. p. 145), I showed how to obtain in a simple way the surface-conditions satisfied by the magnetic potential at a thin sheet of metal of finite conductivity of any form, in which currents are induced by the motion of magnets on either side of the sheet.

In the case of a *cylindrical* sheet of small thickness c, conductivity C, and radius a, the magnetic potentials Ω_1 and Ω_2 inside and outside the sheet were found to be connected by surface-conditions (at $r=a$) which may be written

$$\frac{d}{dt}\left(r\frac{d\Omega_1}{dr}\right)=\frac{d}{dt}\left(r\frac{d\Omega_2}{dr}\right)=\frac{\mathrm{R}}{2}\frac{d}{dr}r\frac{d}{dr}(\Omega_2-\Omega_1), \quad . \quad (1)$$

involving the condition $d\Omega_1/dr=d\Omega_2/dr$; where

$$\mathrm{R}=\frac{1}{2\pi\mathrm{C}c},$$

so that R is the velocity with which the trails of images in a plane sheet of the same thickness and conductivity would move away from the sheet.

For a *spherical* sheet we found in like manner

$$\frac{d}{dt}\left(r^2\frac{d\Omega_1}{dr}\right)=\frac{d}{dt}\left(r^2\frac{d\Omega_2}{dr}\right)=\frac{\mathrm{R}}{2}\frac{d}{dr}r^2\frac{d}{dr}(\Omega_2-\Omega_1), \quad . \quad (2)$$

involving, as before, the condition $d\Omega_1/dr=d\Omega_2/dr$.

Before determining the images which represent the effect of the currents induced by the introduction of a given pole, magnet, or current in the presence of such sheets, it will be convenient to think, for a moment, of the images which occur in hydrodynamical problems relating to the cylinder and sphere. In two-dimensional fluid motion, the image of a line-source outside and parallel to the axis of a cylinder is an equal

* Read February 11, 1898.

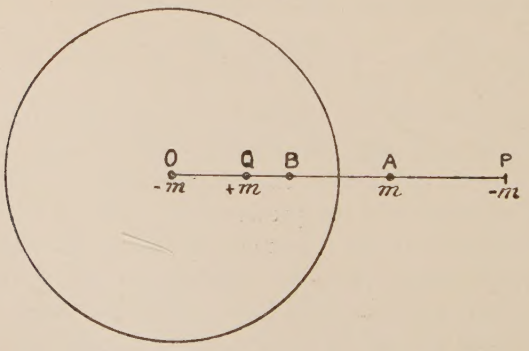
line-source through the inverse point together with an equal line-sink through the centre (*i. e.* along the axis); similarly the image of a line-source inside the sphere together with an equal line-sink along the axis is an equal line-source through the inverse point. The image of a vortex-filament parallel to the axis of a cylinder, whether inside or outside the cylinder, is an equal and opposite vortex through the inverse point. In the case of a sphere, the image of a doublet whose axis passes through the centre is a certain doublet at the inverse point, whether the original doublet be inside or outside the sphere. The image of a *source* in a sphere, on the other hand, is more complicated, consisting not only of a source but also of a sink extending to the centre if the original source was outside the sphere, and to infinity if inside.

We may therefore expect by analogy that the images in a spherical sheet due to the motion or generation of magnetic *poles* in its neighbourhood will be less simple than those produced by *magnets* whose axes pass through the centre of the sphere; and such we shall find to be actually the case.

Image due to Generation of a Line of Poles parallel to the Axis outside a Cylindrical Sheet.

2. We first consider the two-dimensional problem in which the currents are induced by the generation or destruction of a linear distribution of magnetism parallel to the axis of the cylinder at A (fig. 1) in the line whose polar (cylindrical)

Fig. 1.



coordinates are $r=b$, $\theta=0$, where b is greater than a the

radius of the cylinder. To cover all cases we commence by assuming at the outset that the line-density of this line of poles, as it may be called, is $f(t)$ a function of the time.

The magnetic potential of this distribution at the point (r, θ) is therefore

$$\Omega_0 \text{ (say)} = \mu f(t)\{\text{const.} - \log \surd(r^2 + b^2 - 2rb\cos\theta)\},$$

where μ, the magnetic permeability of the dielectric, will be taken to be unity, or included under $f(t)$.

The above potential may, for values of r less than b, be expanded in the form

$$\Omega_0 = f(t)\left\{\text{const.} - \log b + \Sigma\frac{1}{n}\frac{r^n}{b^n}\cos n\theta\right\}. \quad . \quad (3)$$

Consistently with the surface condition of continuity of magnetic induction, we can assume for the potentials of the induced currents inside and outside the cylinder expressions of the form

$$\Omega' = -\Sigma\frac{A_n}{n}\frac{r^n}{a^n}\cos n\theta \text{ (inside)}, \quad . \quad . \quad . \quad (4)$$

$$\Omega'' = \Sigma\frac{A_n}{n}\frac{a^n}{r^n}\cos n\theta \text{ (outside)}, \quad . \quad . \quad . \quad (5)$$

since these make $d\Omega'/dr = d\Omega''/dr$ when $r = a$; here the coefficients A_n are functions of the time t.

Substituting the expressions thus obtained, $\Omega_0 + \Omega'$ for Ω_1, and $\Omega_0 + \Omega''$ for Ω_2 in the surface-conditions (1), we have, on equating coefficients of $\cos n\theta$,

$$\frac{dA_n}{dt} + n\frac{R}{a}A_n = f'(t)\frac{a^n}{b^n};$$

$$\therefore \quad \frac{d}{dt}\left\{A_n e^{ntR/a}\right\} = f'(t)e^{ntR/a}\frac{a^n}{b^n};$$

$$\therefore \quad A_n = \int_{-\infty}^{t} f'(\tau)e^{-n(t-\tau)R/a}d\tau \cdot \frac{a^n}{b^n},$$

the limits of integration being t and $-\infty$ if we have to take account of changes that have been taking place from an infinite time back.

To analyse the images, consider now the case in which a

line of poles of strength m is suddenly generated at A at the time 0, so that

$$f(t)=0 \text{ when } t<0,$$
$$f(t)=m \text{ when } t>0,$$

$f'(t)$ vanishes except when $t=0$, and the time-integral of $f'(t)$ over a very small interval containing the time 0 is equal to m. We thus obtain

$$A_n=me^{-ntR/a}\frac{a^n}{b^n}.$$

Substituting in the expressions for Ω' and Ω'' we find

$$\Omega'=-m\Sigma\frac{1}{n}\frac{r^n}{b^n e^{ntR/a}}\cos n\theta$$

$$=-m\left[\log be^{tR/a}-\log\{r^2+(be^{tR/a})^2-2rbe^{tR/a}\cos\theta\}^{\frac{1}{2}}\right] . \quad (6)$$

$$\Omega''=m\Sigma\frac{1}{n}\frac{a^{2n}}{b^n e^{ntR/a}}\cos n\theta$$

$$=m\left[\log r-\log\left\{r^2+\left(\frac{a^2}{be^{tR/a}}\right)^2-2r\left(\frac{a^2}{be^{tR/a}}\right)\cos\theta\right\}^{\frac{1}{2}}\right]. \quad (7)$$

Hence we have the following results :—

(i.) At points inside the conductor, the magnetic potential due to the induced currents initially neutralizes that produced by the sudden generation of the line of poles at A, and is equivalent to that of a line of poles (P, fig. 1) of constant strength $-m$, starting at the point A and moving away from the centre so that its distance from the centre at time t is $e^{tR/a}b$.

(ii.) At points outside the conductor the magnetic potential is equivalent to that of two lines of poles, one of constant strength $-m$ at O the centre, and the other (Q) of strength $+m$, starting at B the inverse point of A $(a^2/b, 0)$, and moving towards the centre so that at time t its distance from the centre is $e^{-tR/a}a^2/b$. We observe that P and Q, the positions of the inside and outside images, move so as always to remain inverse points.

The velocities with which the images P, Q move away from and towards O respectively are R/a times their distances from O, and their accelerations are R^2/a^2 times these distances respectively.

3. The current function at any point M of the cylinder is

given by

$$\phi = \frac{1}{4\pi}(\Omega_2 - \Omega_1) + \text{a constant};$$

therefore ϕ differs by a constant from

$$-\frac{m}{4\pi}(\log \mathrm{PM} + \log \mathrm{QM}).$$

The arbitrary constants in ϕ may be functions of the time, but they do not affect the distribution of currents in the cylinder, and therefore need not be considered. Since P and Q are inverse points

$$\mathrm{PM} : \mathrm{QM} = \mathrm{OP} : a = a : \mathrm{OQ} = be^{\mathrm{R}t/a} : a;$$

log PM and log QM therefore differ by a term which may be included in the arbitrary constant of ϕ, so that for determining the currents we may take

$$\phi = -\frac{m}{2\pi}\log \mathrm{PM}, \quad \text{or } \phi = -\frac{m}{2\pi}\log \mathrm{QM},$$

whichever is most convenient.

The current at M is evidently parallel to the axis (since ϕ is independent of the third cylindrical coordinate z), and the intensity of the current is equal to

$$\frac{d\phi}{ad\theta} = -\frac{m}{2\pi}\frac{\mathrm{OP}\sin\theta}{\mathrm{PM}^2} = -\frac{m}{2\pi}\frac{\mathrm{OQ}\sin\theta}{\mathrm{QM}^2}.$$

Images due to Generation of Lines of Poles inside the Cylindrical Sheet.

4. If we were to suppose the currents to be induced by a line of poles of strength $f(t)$ at the point $(b, 0)$ where $b < a$, we should have to expand the potential in descending powers of r in order to obtain a convergent series at the surface of the sheet. We should thus obtain for the inducing potential

$$\Omega_0 = f(t)\left\{\text{const.} - \log r + \Sigma\frac{1}{n}\frac{b^n}{r^n}\cos n\theta\right\}.$$

Now the portion

$$f(t)\{\text{const.} - \log r\}$$

represents the potential of a line of poles of strength $f(t)$ along the axis, for investigating the effects of which the

image method is inconvenient. To remove these terms we shall suppose the inducing system to consist of a line of poles through the point $(b, 0)$ together with an equal and opposite line of poles along the axis: in other words of a line of magnets having their positive poles $f(t)$ at $(b, 0)$ and their negative poles $-f(t)$ at the axis. The potential of such a distribution expands where $r > b$ in the form

$$\Omega_0 = f(t)\Sigma\frac{1}{n}\frac{b^n}{r^n}\cos n\theta. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The analysis is almost identical with that of the preceding case, and passing to the case when

$$f(t) = 0 \text{ when } t < 0,$$
$$f(t) = m \text{ when } t > 0,$$

the following results may be obtained, taking B (fig. 1) to now represent the point $(b, 0)$.

(i.) If lines of poles of strengths m and $-m$ be instantaneously generated at B and the axis respectively, the images representing the magnetic potential of the induced currents outside the cylinder after a time t are lines of poles of strengths $-m$ and $+m$ at Q and the axis, where $OQ = e^{-tR/a}b$. Hence the *total* magnetic potential at outside points due to the poles and the induced currents at time t is that of lines of poles of strengths $+m$ and $-m$ at B and Q respectively.

(ii.) At points inside the cylinder, the magnetic potential due to the induced currents alone is that which would be produced by a line of poles of strength $+m$ at P, where P is the inverse point of Q, so that $OP = e^{tR/a}a^2/b$.

Images due to Generation of Straight Currents Parallel to the Axis of the Cylindrical Sheet.

5. The cases where the currents are induced by the sudden generation of a straight current parallel to the axis of the cylinder probably have a more practical physical interest than those considered in the previous articles. If we suppose all the lines of poles of strengths $\pm m$ of the preceding sections to be replaced by currents of intensities $\pm I$ the results regarding the images will be identically the same as before. To satisfy ourselves that such is the case we have only to notice that while the magnetic potential of a line

of poles of line-density m at $(b, 0)$ is the real part of the expression

$$m \log (re^{i\theta} - b),$$

the magnetic potential of a current perpendicular to the plane of reference and of intensity $\frac{1}{2}m$ is the imaginary part of the same expression. Where $r < b$, the potential of the current expands therefore in the series

$$m\Sigma \frac{1}{n} \frac{r^n}{b^n} \sin n\theta,$$

and where $r > b$ the potential of a current $\frac{1}{2}m$ at $(b, 0)$ together with a return current $-\frac{1}{2}m$ down the axis expands in the series

$$m\Sigma \frac{1}{n} \frac{b^n}{r^n} \sin n\theta.$$

These expressions differ from those used in the previous articles by the substitution of sines for cosines only, and it follows that when we make similar changes in the assumed expressions for the potentials of the induced currents, the work of substituting in the boundary conditions will be unaltered. For convenience, however, it will be well to again state the final conclusions, which are as follows :—

(i.) If a current of strength I be suddenly generated parallel to the axis of the cylinder at A *outside* the cylinder, the image representing the effect of the induced currents for inside points at time t will be a current of strength $-$I at P, and the images representing their effect for outside points will be a current $+$I at Q and a return current $-$I in the axis, where $OP = e^{tR/a}OA$, and Q is the inverse point of P, so that $OQ = e^{-tR/a}a^2/OA$.

(ii.) If a current of strength I be suddenly generated at B *inside* the cylinder, accompanied by a return current $-$I in the axis, the image representing the effect of the induced currents for outside points will be a current I at P, and the images representing their effect for inside points will be currents $-$I at Q, and I in the axis, where $OQ = e^{-tR/a}OB$, and P is the inverse point of Q, so that $OP = e^{tR/a}a^2/OB$.

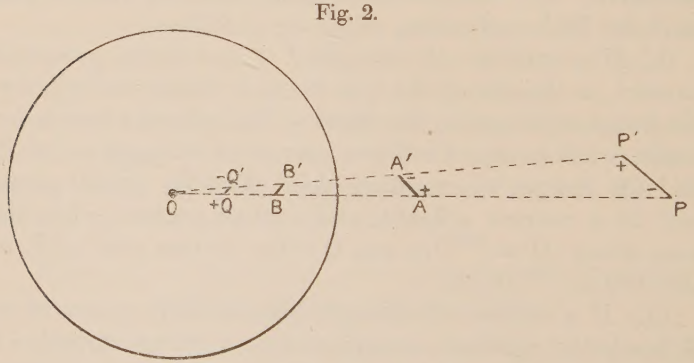
The effect of *breaking* a current of intensity I will of course be the same as that of superposing a current $-$I and the images will be equal and opposite to those above obtained.

Images due to sudden Generation of Lines of Magnets, and Moving Poles.

6. It is now easy to build up the images due to the sudden generation or destruction of lines of magnets parallel to the axis of the cylinder, the motion of lines of poles and so forth. The methods are almost identical with those used for plane sheets, and it will be sufficient here to call attention to one or two salient features of the images in cylindrical sheets. As in the previous articles we shall take A, B to be any two inverse points inside and outside the cylinder, P and Q two other inverse points such that $OP = e^{tR/a}OA$, $OQ = e^{-tR/a}OB$, and similar relations will be supposed to hold when the letters are accented.

(i.) If a line of magnets parallel to the axis at AA′ be generated suddenly, having their positive poles at A and their negative poles at A′ (fig. 2), the image at time t for inside points will be the oppositely magnetized line PP′, and for outside points the line QQ′ having its positive magnetism at Q. From the properties of similar triangles and of inversion, PP′ is

Fig. 2.



parallel and QQ′ is antiparallel to AA′, also, since the strengths of the poles are constant, the magnetic moments of the images are to one another and to that of the inducing system as PP′ : QQ′ : AA′, that is as OP : OQ′ : OA or, in the limiting case (when Q and Q′ are infinitely near together), as the distances OP : OQ : OA.

Hence if M be the magnetic moment generated in AA′,

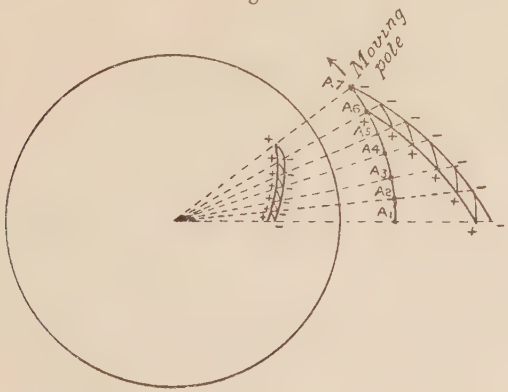
those of the images PP′ and QQ′ at time t are $-Me^{tR/a}$ and $Me^{-tR/a}a^2/b^2$, where $b=OA$.

(ii.) Similarly, if a line of magnets of moment M be suddenly generated at BB′ *inside* the cylinder, the images at time t for outside points will be the oppositely magnetized line QQ′ of magnetic moment $-Me^{-tR/a}$, and for inside points the magnetized line PP′ having its positive poles at P and of magnetic moment $Me^{tR/a}a^2/b^2$, b being now taken $=OB$.

7. If a line of poles of strength m suddenly changes position from A′ to A the effect is equivalent to the superposition of $-m$ at A′ and $+m$ at A, and is thus equivalent to the generation of a line of magnets in the direction AA′, the images of which have just been found.

Fig. 3 shows the application of this method to construct the trails of images of a line of poles revolving uniformly about the centre and passing in succession through the positions A_1, A_2, A_3, A_4, ..., at equal small intervals of time, the actual trails are of course the limits of the series of moving poles thus plotted out. By supposing the line of poles to

Fig. 3.



remain fixed and the cylinder to rotate, we get a kind of cylindrical two-dimensional Arago's disk, and it is to be noticed that the trails of images lie along equiangular spirals (as may readily be verified), the intensity of magnetization of the trail decreasing toward the centre and increasing outward and being, in fact, proportional to the distance from the centre.

Part III. Spherical Sheets.

Image due to Generation of a Small Magnet whose Axis is Radial outside a Spherical Sheet.

8. We now consider the problem presented by the currents induced in a spherical sheet of radius a when a magnet is generated at a distance b from the centre, the axis of the magnet passing through the centre of the sphere. Taking this axis as the axis of zonal spherical harmonics, and starting in the first instance with a magnet of variable moment $f(t)$, (the positive pole being furthest from the centre), the magnetic potential Ω_0 at a point (r, θ) is given by

$$\Omega_0 = f(t) \frac{d}{db} \frac{1}{\sqrt{(r^2+b^2-2rb\cos\theta)}}.$$

When $r<b$ this becomes

$$\Omega_0 = f(t) \frac{d}{db} \Sigma_{n=0}^{n=\infty} \frac{r^n}{b^{n+1}} P_n(\cos\theta)$$

$$= -f(t) \Sigma_{n=0}^{n=\infty} \frac{(n+1)r^n}{b^{n+2}} P_n; \quad . \quad . \quad . \quad (9)$$

and when $r>b$ we have in like manner

$$\Omega_0 = f(t) \frac{d}{db} \Sigma_{n=0}^{n=\infty} \frac{d}{db} \frac{b^n}{r^{n+1}} P_n(\cos\theta)$$

$$= f(t) \Sigma_{n=1}^{n=\infty} \frac{nb^{n-1}}{r^{n+1}} P_n. \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Considering the case where the magnet is outside the sphere, we must take the first expression, since at the surface of the sphere $r<b$.

Let the potentials due to the induced currents at points inside and outside the sphere be assumed of the form

$$\Omega' = \Sigma(n+1)A \frac{r^n}{a^{n+1}} P_n \text{ (inside)}, \quad . \quad . \quad (1_1)$$

$$\Omega'' = -\Sigma n A_n \frac{a^n}{r^{n+1}} P_n \quad \text{(outside)}, \quad . \quad . \quad (12)$$

where the coefficients A_n are functions of the time t; the assumed forms being such as to satisfy the surface-condition $d\Omega'/dr = d\Omega''/dr$.

Substituting the expressions for the potential, $\Omega_0+\Omega'$ and $\Omega_0+\Omega''$ for Ω_1 and Ω_2 in the surface-conditions (2),

$$\frac{d}{dt}\left(r^2\frac{d\Omega_1}{dr}\right)=\frac{d}{dt}\left(r^2\frac{d\Omega_2}{dr}\right)=\frac{\mathrm{R}}{2}\frac{d}{dr}\,r^2\frac{d}{dr}(\Omega_2-\Omega_1),$$

we find, on equating coefficients of P_n,

$$\frac{d\mathrm{A}_n}{dt}+\frac{(2n+1)\mathrm{R}}{2a}\mathrm{A}_n=f'(t)\frac{a^{n+1}}{b^{n+2}},$$

$$\therefore\quad \frac{d}{dt}\{\mathrm{A}_n e^{(2n+1)t\mathrm{R}/2a}\}=f'(t)e^{(2n+1)t\mathrm{R}/2a}\frac{a^{n+1}}{b^{n+2}},$$

$$\therefore\quad \mathrm{A}_n=\int_{-\infty}^{t}f'(\tau)e^{-(2n+1)(t-\tau)\mathrm{R}/2a}\,d\tau\,.\,\frac{a^{n+1}}{b^{n+2}}.$$

Now let the inducing system be a magnet of moment M suddenly generated at the time $t=0$, so that

$$f(t)=0 \text{ when } t<0,$$
$$f(t)=\mathrm{M} \text{ when } t>0.$$

Then by a similar artifice to that used for the cylindrical sheet, we have

$$\mathrm{A}_n=\mathrm{M}e^{-(2n+1)t\mathrm{R}/2a}\frac{a^{n+1}}{b^{n+2}}.$$

Substituting in the potentials of the induced currents, we have

$$\Omega'=\mathrm{M}\Sigma(n+1)e^{-(2n+1)t\mathrm{R}/2a}\frac{r^n}{b^{n+2}}\mathrm{P}_n$$

$$=\mathrm{M}e^{3t\mathrm{R}/2a}\Sigma\frac{(n+1)r^n}{(e^{t\mathrm{R}/a}b)^{n+2}}\mathrm{P}_n, \quad . \quad . \quad . \quad . \quad (13)$$

$$\Omega''=-\mathrm{M}\Sigma ne^{-(2n+1)t\mathrm{R}/2a}\frac{a^{2n+1}}{b^{n+2}r^{n+1}}\mathrm{P}_n.$$

$$=-\mathrm{M}\frac{a^3}{b^3}e^{3t\mathrm{R}/2a}\Sigma\frac{n(e^{-t\mathrm{R}/a}a^2/b)^{n-1}}{r^{n+1}}\mathrm{P}_n. \quad . \quad (14)$$

Comparing the first of these results with (9) and the second with (10), we have the following conclusions:—

(i.) At time t after the generation of the magnet, the potential due to the induced currents is, at points inside the sphere, equivalent to that produced by a magnet of moment $-\mathrm{M}e^{3t\mathrm{R}/2a}$ at a distance from the centre of $e^{t\mathrm{R}/a}b$.

(ii.) At points outside the sphere, the potential is equivalent to that of a magnet of moment $-\mathrm{M}e^{-3t\mathrm{R}/2a}a^3/b^3$ at a distance from the centre of $e^{-t\mathrm{R}/a}a^2/b$. The positions P, Q occupied by these images at time t are always inverse points.

It is easily seen that the intensities of the images at P and Q are to each other and to that of the inducing magnet as $\mathrm{OP}^{\frac{3}{2}} : \mathrm{OQ}^{\frac{3}{2}} : \mathrm{OA}^{\frac{3}{2}}$, that is in the sesquiplicate ratio of their distances from the centre.

Images due to Generation of a Small Magnet inside a Spherical Sheet.

9. If $b<a$ or the magnet is generated within the sphere, we must expand the potential in the second form (10) in order to substitute in the surface-conditions. When this is done, it will be found that

$$\mathrm{A}_n = \mathrm{M}e^{-(2n+1)t\mathrm{R}/2a}\frac{b^{n-1}}{a^n}$$

and the conclusions finally arrived at are as follows :—

(i.) If a small magnet of moment M be instantaneously generated at a point B inside the sphere at a distance b from the centre, the axis of the magnet being radial ; then the image representing the magnetic potential of the induced currents outside the sphere at time t is a magnet of moment $-\mathrm{M}e^{-3t\mathrm{R}/2a}$ at a point Q such that $\mathrm{OQ}=be^{-t\mathrm{R}/a}$.

(ii.) At points inside the cylinder the potential is that which would be produced by a magnet of moment $-\mathrm{M}e^{3t\mathrm{R}/2a}a^3/b^3$ at a distance from the centre of $e^{t\mathrm{R}/a}a^2/b$.

As before, the intensities of the several magnets vary as the $\frac{3}{2}$ power of their distance from the centre.

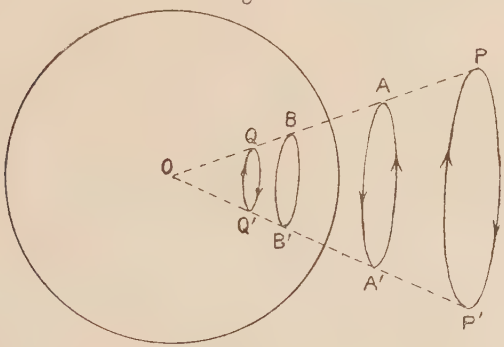
Images due to Generation of Circular Currents coaxial with a Spherical Sheet, &c.

10. A circular current whose axis passes through the centre of the sphere is equivalent to a magnetic shell forming a spherical cap on the surface of a concentric sphere, this cap being bounded by the current. It is thus evident that the images due to the sudden generation of such a current are also circular currents, which move in the manner described in the preceding sections. There is no need to restrict the results to

circular currents as the same arguments are equally applicable when the inducing current flows in any closed curve traced on a spherical surface concentric with the sheet, the images in this case being currents in similar curves. It remains to state the relation between the intensities of the currents in the images and that of the inducing current.

(i.) Let AA′ (fig. 4) be the wire in which an inducing

Fig. 4.



current of intensity I is suddenly generated *outside* the sheet. Then at time t the image for inside points will be a current in PP′ and for outside points a current in QQ′, where $OP = e^{tR/a}OA$ and $OQ = e^{-tR/a}a^2/OA$. Suppose the currents replaced by their equivalent magnetic shells on spherical surfaces centred at O, and let corresponding elements of these shells be taken. The magnetic moments of the elements of PP′, QQ′, and AA′ are respectively as $OP^{\frac{3}{2}} : OQ^{\frac{3}{2}} : OA^{\frac{3}{2}}$; and the areas of these elements are as $OP^2 : OQ^2 : OA^2$; hence the strengths of the shells are as $OP^{-\frac{1}{2}} : OQ^{-\frac{1}{2}} : OA^{-\frac{1}{2}}$, and the current-intensities are in the same ratio. Thus finally, if I′ and I″ be the intensities of the image-currents at time t in PP′ and QQ′, we have

$$I' = -\left(\frac{OP}{OA}\right)^{-\frac{1}{2}} I = -e^{-tR/2a}\, I,$$

$$I'' = -\left(\frac{OQ}{OA}\right)^{-\frac{1}{2}} I = -e^{tR/2a}\,\frac{b}{a}\, I.$$

It will be noticed that I″ increases as its circuit QQ′ approaches the centre, while I′ decreases as its circuit PP′ recedes to infinity.

(ii.) Next taking the inducing current I to be suddenly generated in BB′ *inside* the sphere, and taking $OB=b$, $OQ=e^{-tR/a}b$, and $OP=e^{tR/a}a^2/b$, we find that at time t, the image representing the potential of the induced currents at points outside the sphere is a current in QQ′ of intensity

$$-e^{tR/2a}\,I,$$

and at points inside the sphere a current in PP′ of intensity

$$-e^{-tR/2a}\,\frac{b}{a}\,I.$$

Images due to Generation of Magnetic Pole outside a Spherical Sheet.

11. A magnetic pole m generated at time 0 outside a sphere at the point $r=b$ where $b>a$ might be regarded as equivalent to a line of elements uniformly magnetized in the direction of the radius-vector extending along that radius from $r=b$ to infinity ; and similarly a positive pole m inside the sphere at $r=b$ when $b<a$ together with a negative pole $-m$ at the centre might be regarded as equivalent to a line of elements uniformly magnetized in the line joining these points. In this way the images might be obtained by integration. Practically as short a way of obtaining the result as any is by utilizing the work of §§ 8, 9, taking into account the differences introduced into the coefficients of the various harmonics when for the inducing potential of a magnet there is substituted that of a pole.

When the pole is outside the sphere the inducing potential after its generation becomes

$$\Omega_0=m\Sigma\frac{r^n}{b^{n+1}}\,P_n\quad(r<b).\quad .\quad .\quad .\quad .\quad (15)$$

The coefficient of P_n is $-b/(n+1)$ of the corresponding coefficient for a magnet of moment m.

(i.) Hence we find for the potentials of the induced currents at points inside the sphere at time t

$$\Omega'=-m\Sigma e^{-(2n+1)tR/2a}\frac{r^n}{b^{n+1}}P_n$$

$$=-me^{tR/2a}\Sigma\frac{r^n}{(be^{tR/a})^{n+1}}\,P_n.\quad .\quad .\quad .\quad (16)$$

This is the potential of a pole of intensity $-me^{tR/2a}$ at a distance from the centre of $be^{tR/a}$.

(ii.) For points inside the sphere the corresponding potential is

$$\Omega'' = m\Sigma \frac{n}{n+1} e^{-(2n+1)tR/2a} \frac{a^{2n+1}}{b^{n+1} r^{n+1}} P_n$$

$$= m\frac{a}{b} e^{-tR/2a} \Sigma \frac{(e^{-tR/a}a^2/b)^n}{r^{n+1}}$$

$$= m\frac{a}{b} e^{-tR/2a} \Sigma \frac{n}{n+1} \frac{(e^{tR/a}a^2/b)^n}{r^{n+1}} P_n.$$

Let $\rho = e^{-tR/a}a^2/b$. Then

$$\Sigma \frac{n}{n+1} \frac{\rho^n}{r^{n+1}} P_n = \Sigma \frac{\rho^n}{r^{n+1}} P_n - \frac{1}{\rho} \Sigma \frac{\rho^{n+1}}{(n+1) r^{n+1}} P_n$$

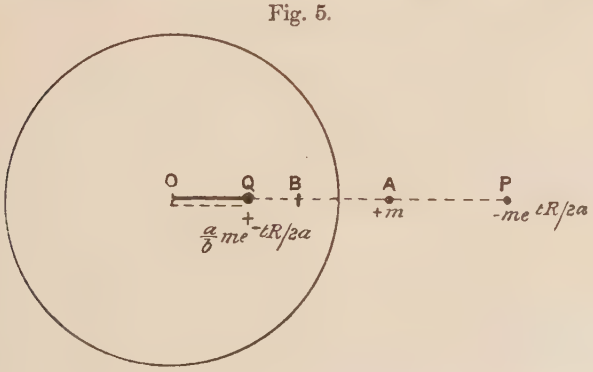
$$= \Sigma \frac{\rho^n}{r^{n+1}} P_n - \frac{1}{\rho} \int_0^\rho \Sigma \frac{\rho'^n}{r^{n+1}} P_n \, d\rho'$$

$$= \frac{1}{\sqrt{(r^2 - 2r\rho\cos\theta + \rho^2)}} - \frac{1}{\rho} \int_0^\rho \frac{d\rho'}{\sqrt{(r^2 - 2r\rho'\cos\theta + \rho'^2)}}$$

$$\therefore \Omega'' = \frac{me^{-tR/2a}a/b}{\sqrt{(r^2 - 2r\rho\cos\theta + \rho^2)}} - \frac{m}{a} e^{tR/2a} \int_0^\rho \frac{d\rho'}{\sqrt{(r^2 - 2r\rho'\cos\theta + \rho'^2)}}. \quad . \quad . \quad . \quad (17)$$

The first part represents the potential of a pole of strength $me^{-tR/2a}a/b$ at a distance ρ from the centre, *i. e.* at the point Q of the previous sections, and the second represents the potential of a line-distribution of magnetism of uniform linear

Fig. 5.



density $-me^{tR/2a}/a$ extending from the centre to Q ; this being

equal to the strength of the pole divided by ρ or OQ, it follows that the total quantity of magnetism on this line OQ is equal and opposite to that in the pole Q (fig. 5). This distribution of images is related to the image on the opposite side in the same manner as the hydrodynamic image of a source in a sphere is related to the source, except in the matter of algebraic sign. The reason for the resemblance is that the two potentials here have to make $d\Omega'/dr = d\Omega''/dr$ at the surface, while in the hydrodynamic problem the portions of the normal velocity components $d\phi/dr$ due to the source and its image have to cancel one another.

Images due to Generation of Magnetic Poles inside a Spherical Sheet.

12. If a pole m be generated inside a sphere at the point $r=b$ where $b<a$, together with an equal and opposite pole $-m$ at the centre, the inducing potential takes the form

$$\Omega_0 = m\Sigma_{n=1}^{n=\infty} \frac{b^n}{r^{n+1}} P_n. \quad . \quad . \quad . \quad . \quad . \quad (18)$$

(i.) The induced potential outside the sphere takes the form

$$\Omega'' = -me^{-tR/2a}\Sigma_{n=1}^{n=\infty} \frac{(be^{-tR/a})^n}{r^{n+1}} P_n. \quad . \quad . \quad . \quad . \quad (19)$$

This is the potential of a pole of intensity $-me^{-tR/2a}$ at a distance from the centre of $be^{-tR/a}$, together with an equal and opposite pole at the centre.

(ii.) The induced potential inside is

$$\Omega' = m\Sigma \frac{n+1}{n} e^{-(2n+1)tR/2a} \frac{r^n b^n}{a^{2n+1}} P_n.$$

If $\rho = e^{tR/a}a^2/b$, this becomes

$$\Omega' = m\frac{a}{b} e^{tR/2a}\Sigma \frac{n+1}{n} \frac{r^n}{\rho^{n+1}} P_n$$

$$= m\frac{a}{b} e^{tR/2a} \left\{ \Sigma \frac{r^n}{\rho^{n+1}} P_n - \frac{1}{\rho} \int_\rho^\infty \Sigma \frac{r^n}{\rho'^{n+1}} d\rho' \right\}. \quad . \quad (20)$$

The images thus consist of a positive pole of strength $me^{tR/2a}a/b$ at a distance from the centre ρ equal to $e^{tR/a}a^2/b$ together with a line-distribution of magnetism of linear

density $-me^{-tR/2a}/a$ extending from distance ρ to infinity. This linear density is equal to the strength of the pole-image divided by its distance ρ.

Other Images in a Spherical Sheet.

13. When the inducing potential is due to a small *magnet whose axis does not pass through the centre* of the sphere we could of course find the images by considering the positive and negative poles separately. If the axis of the magnet makes an angle α with the radius-vector the simplest plan would be to replace it by two magnets of moment $M \cos \alpha$ and $M \sin \alpha$ along and perpendicular to the radius.

14. The images due to *a pole of constant strength moving along the radius-vector* and initially at rest will be trails of magnets which can be worked out by employing the simpler images of §§ 10 and 11. If, initially, the pole were *suddenly generated*, we should have to introduce the more complicated images of §§ 11 and 12, but this may be avoided by considering the problem in which the pole is supposed to have remained at rest from an infinite time past (or, at any rate, so long that all currents in the sheet have died out), and investigating only the effects due to its motion. As usual, any small displacement may be represented by putting down a magnet of which one pole cancels the moving pole in its old position, while the other is identical with the moving pole in its new position. If the pole moves in a direction other than radial, the more complicated images of §§ 11 and 12 will of course be introduced.

General Conclusions.

1. The phenomena of two-dimensional induction in cylindrical sheets and of induction in spherical sheets due to a sudden disturbance in the magnetic field can be represented by moving trails of images which are generally not much more complicated than those obtained with a plane sheet.

2. The images in every case start from the source of disturbance and its inverse, and move away from the sheet radially with velocities varying directly as the distance, the constant of variation being such as to make the velocity at the surface of the sheet become equal to R, the corresponding

velocity for a plane sheet of the same thickness and conductivity. The image which initially coincides with the source of disturbance initially neutralizes its effect on the opposite side of the sheet.

3. Where the source of disturbance is a distribution of poles or an unclosed current *inside* the sheet it is necessary from physical considerations to place equal and opposite poles at the centre or to close the circuit by axial return currents, and this is also essential for the success of the image method.

4. The relations between the intensity of the source of disturbance and those of the images will readily be found to be in every case expressible in terms of their distances from the centre. The results expressed in this way may be tabulated as follows :—

Sheet.	Inducing disturbance.	Law of variation.
Cylinder.	Line of poles parallel to axis.	Constant.
,,	Current parallel to axis.	,,
,,	Line of magnets.	$\propto$ distance.
Sphere.	Magnet radially magnetized.	$\propto$ (distance)$^{\frac{3}{2}}$.
,,	Concentric magnetic shell. / Current on concentric sphere.	$\propto$ (distance)$^{\frac{1}{2}}$.
,,	Magnetic pole (*with exception*).	$\propto$ (distance)$^{\frac{1}{2}}$.

5. The last-named exceptional case is that of a spherical sheet when the disturbance is due to the generation of a pole, and it is required to represent by images the potential of the currents on the same side as the disturbing pole, the images thus being on the opposite side of the sheet. Here, in addition to the single pole whose intensity and position are given by the above statements, the image consists of a line-distribution of magnetism of opposite sign, extending from that pole along the radius away from the sheet, and having its line-density equal to the quantity of magnetism in the pole or image divided by its distance from the centre.

6. When the source of disturbance is in motion, the trails of images can be built up synthetically as in the case of a plane sheet. If the source of disturbance is rotating

uniformly about the centre of the cylinder or sphere, the curves described by the trails of images will be equiangular spirals.

Discussion.

Dr. S. P. Thompson said the method and the results obtained would find useful application in the solution of many allied problems.

XVI. *On the Properties of Liquid Mixtures.*—Part II. *By* R. A. Lehfeldt, *Professor of Physics at the E. London Technical College**.

In a previous article (Phil. Mag. (5) vol. xl. p. 398) an attempt was made to follow out the consequences of a certain thermodynamic relation between the composition of a liquid mixture, and that of the vapour in equilibrium with it, and the saturation pressure of the system. Experiments were there described on mixtures of benzene with ethyl and methyl acetates in which a small fraction of each mixture was distilled, as nearly as possible at a constant temperature, and the distillate analysed ; these led to an empirical expression connecting the composition of liquid and vapour. Experiments were afterwards made to determine the vapour-pressure of similar mixtures by the dynamic method, but they led to unsatisfactory results owing to decomposition of the esters on prolonged boiling. I was able to resume the work in 1897, and then chose more stable compounds to work on, viz., benzene and toluene mixed with carbon tetrachloride, as types of normal organic compounds, and benzene and toluene mixed with ethyl alcohol, as type of a so-called "associated" liquid. The experiments to be described here were carried out at the Davy-Faraday laboratory, which the managers of the Royal Institution very obligingly placed at my service.

Two other papers on the same subject, which appeared about the same time as my earlier one, call for some notice.

* Read March 11, 1898.

These are by C. E. Linebarger (Journ. of Amer. Chem. Soc. vol. xvii.) and by M. Margules (*Wien. Ber.* vol. civ.).

Linebarger made measurements of the vapour-pressures of certain liquid mixtures, and of the way in which the two components shared the pressure between them. He did not attempt to relate his results to the deductions of thermodynamics, but merely to obtain empirical generalizations. The method he adopted is at first sight a very promising one, as it allows of the determination of both total and partial pressures in the same experiment; it consisted in drawing a measured volume of air through the liquid mixture, and analysing by combustion the vapour which the air carried away with it. The method has been applied successfully to find the vapour-pressure of aqueous solutions of low volatility, but it is not so suitable to the present case, as in Linebarger's experiments the vapour-pressure was sometimes as high as 300 mm., and the inaccuracies of it appear to increase out of proportion to the pressure to be measured. Linebarger tested his apparatus by preliminary measurements of pure liquids: finding for instance, for ethyl iodide 199 mm. against Regnault's 206, and for chloroform 290·1 against 301·1. This he calls a "most excellent correspondence." I tried the method before reading the account of his results, and found similar discrepancies of one or two per cent., so that it can evidently not be regarded as satisfactorily worked out as yet. It has also the disadvantage of being so slow that it is impossible to get numerous data. I have, therefore, preferred to revert to the better known methods, and nearly all the observations mentioned below were taken by the "dynamic" method.

Linebarger gives an empirical result which may be considered along with the observations contained in this paper: that strictly normal liquids, such as benzene and toluene, have in mixtures a partial pressure simply proportional to the molecular percentage of them present in the liquid. It is important, as on it Linebarger bases a rule for determining the molecular complexity of liquids. To this point we shall have to revert.

Margules's paper consists only of deductions from the previous experiments of others; but it includes a theorem given

in my paper (*loc. cit.*) as well as several others; and in an appendix mentions that all these results have been forestalled in the very systematic thermodynamical studies of Duhem—a fact which I also had overlooked. Margules's chief new contribution is, therefore, the empirical formula he proposes for the relation between composition of liquid and vapour, as will be mentioned below.

Measurements of Vapour-Pressure.

The measurements made form two distinct groups, those of vapour-pressure and those of composition of vapour, which were carried out separately, but on material from the same source, and prepared in identical manner. The materials were as follows :—

Benzene.—Kahlbaum's thiophene-free : its boiling-point was nearly constant, but it was fractionated twice from calcium chloride, and then stood over sodium. B.p. (corrected and reduced to 760 mm.)=80°·0. Vapour-pressure at 50°= 270·9 mm. Density $\frac{18}{4}$=0·8803. μ_D at 18°=1·5024 by spectrometer.

Toluene.—Baird and Tatlock's purest. Fractionated from calcium chloride and stood over sodium. B.p. (corr. and red.) =110°·0. Vapour-pressure at 50°=93·0 mm. Density $\frac{18}{4}$ =0·8667. μ_D at 18°=1·4970.

Carbon Tetrachloride. — Baird and Tatlock. Slightly yellow when obtained ; on fractional distillation it became colourless, and on redistilling the best fraction nearly the whole came over within 0°·1. B.p. 76°·6 (corr. and red.). Vapour-pressure at 50°=310·2 mm. Density $\frac{18}{4}$=1·5979. μ_D at 18°=1·4618.

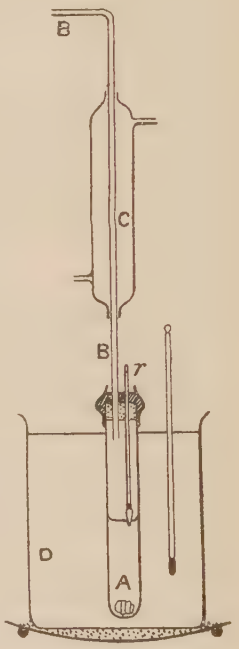
Ethyl Alcohol.—Baird and Tatlock's "absolute." Digested on a water-bath with lime and baryta till a yellow colour appeared ; then distilled with special care to avoid moisture. B.p. (corr. and red.)=78°·3. Vapour-pressure at 50°=219·5 mm. Density $\frac{18}{4}$=0·7929. μ_D at 18°=1·3622.

The density is not so low as it should be according to Mendeléef ; but it is probable that if a purer alcohol had been

obtained it would have absorbed some moisture in the course of the inevitable manipulation of the mixtures. Distillation from sodium was also tried, but did not give any better results.

To measure the vapour-pressure of the mixtures, the dynamic method was adopted. The most interesting part of the apparatus is shown in the accompanying sketch (fig. 1); it consists of a boiling-tube A about 15 cm. × 3, closed at the top by a cork with two holes. Through one passed the thermometer—a short one with milk glass scale 40° to 60° in $\frac{1}{5}$; its bulb was surrounded with a little cotton-wool, and an end of the wool hung down into the liquid, so that the bulb was always moist; if that is the case, and the stem does not touch the side of the boiling-tube, the readings are very trustworthy. Through the cork there passed also a tube B, about 9 mm. wide, surrounded by a short condenser, bent at right angles above, and leading to (1) a mercury manometer, (2) a T-piece, of which the vertical limb passed into a Winchester, serving as a reservoir of air, and containing a little strong sulphuric acid; and the further limb a glass tap. The necks of the boiling-tube and of the winchester were surrounded by a short piece of wide rubber tubing each, and the corks drowned in mercury; the only other joints in the apparatus were the two rubber tubing joints of the manometer and the glass tap; it could be made absolutely air-tight without trouble. On the far side of the glass tap was placed a T-piece leading to a hand air-pump on one side, and on the other into the atmosphere, through a long capillary glass tube which served to reduce the flow of air; the pressure in the apparatus could thus be adjusted with any degree of nicety. The boiling-tube

Fig. 1.



contained about 10 to 15 grams of liquid mixture, previously made up by weighing, and a piece of pumice-stone weighted with copper wire, to make the ebullition steady ; although the thermometer was surrounded by vapour, its readings were never constant unless the liquid was boiling freely. Heat was applied by means of a water-bath, consisting of a two-litre beaker, heated over a sand-tray, with a small flame ; this was provided with a stirrer and thermometer, and kept three or four degrees above the temperature at which the mixture boiled.

An experiment consisted in weighing out a mixture, taking its refractive index by the Pulfrich refractometer, placing in the boiling-tube, and after adjusting temperature and pressures, taking eight observations at temperatures rising from 45° to 55°, and then falling to 45° again, and taking the refractive index of the residue : the refractive index was throughout used as a means of analysis (as in the previous paper, *q. v.*), and it showed that no appreciable change of composition took place during the experiments.

The instruments used were the following :—(i.) A cathetometer (by Fuess) to read the manobarometer with. Its scale was taken as correct; and since readings were only taken to one-tenth of a millimetre, no difficulty was encountered. Pressures were in all cases corrected for temperature of the mercury, and reduced to sea-level in lat. 45°.

(ii.) Five thermometers, of which the most important were "A" of range 40° to 60° in $\frac{1}{5}$, used for the boiling-tube, and "B" of range $-5°$ to $+35°$ in $\frac{1}{5}$, used for the refractometer, both by C. E. Müller. Their errors, which were very small, were determined by comparison with the standards of the Reichsanstalt, nos. 7346 and 7347, belonging to the Davy-Faraday laboratory.

(iii.) Pulfrich refractometer of the old pattern (by Max Wolz). Measurements of the refractive indices of alcohol (1·36), carbon tetrachloride (1·46), and benzene (1·50) were taken by it and a small Schmidt and Haensch spectrometer. The values given by Pulfrich's table were thus found to be too low by 0·00620 for alcohol, 0·00613 for carbon tetrachloride, and 0 00615 for benzene. Accordingly 0·0062 was

added to the number from the table in each case. Measurements of the refractive index of the mixtures were always taken as near to 18° as possible (though unfortunately in the summer the room rose frequently to 21°): the temperature-coefficients were interpolated from the known coefficients of the pure substances (this involves very little error as the coefficients only vary from 0·00040 for alcohol to 0·00064 for benzene), and the readings reduced to the standard temperature.

An accuracy of the order 1 in 1000 was aimed at in the various measurements; but of course the final results are hardly reliable to that extent. To take a typical case, the saturation-pressure to be measured might be about 250 mm.: there is of course no difficulty whatever in reading the height of the mercury-column to one thousandth of that ($\frac{1}{4}$ mm.). The corresponding temperature needs to be known to about $\frac{1}{40}$ degree, since 1° would make about 10 mm. difference in pressure. The thermometer was read to $\frac{1}{50}$ by eye with certainty; and when the apparatus was in good working order, it kept constant to that extent. Again, to change the vapour-pressure by $\frac{1}{1000}$ (temperature constant) would need a change usually as great as $\frac{1}{500}$ or more in the composition; this would correspond to about 2′ of arc on the refractometer—a quantity very easily observed. The composition of the mixture might alter by escape of the vapour past the condenser; this would be discovered by a comparison of the refractive index before and after the boiling—as a matter of fact, the change rarely exceeded 2′: further, it would necessarily alter as some of the substance was present in the form of vapour, but the weight of vapour in the apparatus could not have exceeded $\frac{1}{250}$ of the weight of liquid, and as the composition of the vapour is never 30 per cent. different from that of the liquid, the evaporation of that quantity could not alter the composition of the remainder more than about $\frac{1}{800}$. The degree of accuracy thus anticipated appears to have been realized except in a few unfavourable cases, as will be seen from the following observation, which is given as a specimen, and to show the mode of reduction adopted.

Specimen Observation.

Weighing-bottle 11·3608 grms.
+ toluene . . 17·1159 „ } 52·40 per cent. of CCl_4.
+ CCl_4 . . 23·4510 „

Refractometer before 40° $16\frac{1}{2}'$ at 21°·1. Hence $\mu_{18}=1{\cdot}48817$.
„ after 40° $20\frac{1}{2}'$ at 21°·8. „ $=1{\cdot}48809$.

Height of		Pressure.	Obs. temp.	Corr. temp.	Log pressure.	Log p_{50}.
Barom.	Gauge.					
779·1	614·5	164·6	48°·14	48°·20	2·2164	2·2470
(779·4)	596·8	182·6	50 84	50·92	·2615	59
(779·6)	580·5	199·1	53·18	53·31	·2991	27
779·9	561·3	218·6	55·60	55·79	·3397	12
(779·6)	587·4	192·2	52·24	52·33	·2838	42
(779·4)	608·0	171·4	49·38	49 45	·2340	32
(779·2)	625·3	153·9	46·64	46·69	·1872	36
779·1	641·4	137·7	43·90	43·93	·1389	22
					Mean=	2·2437

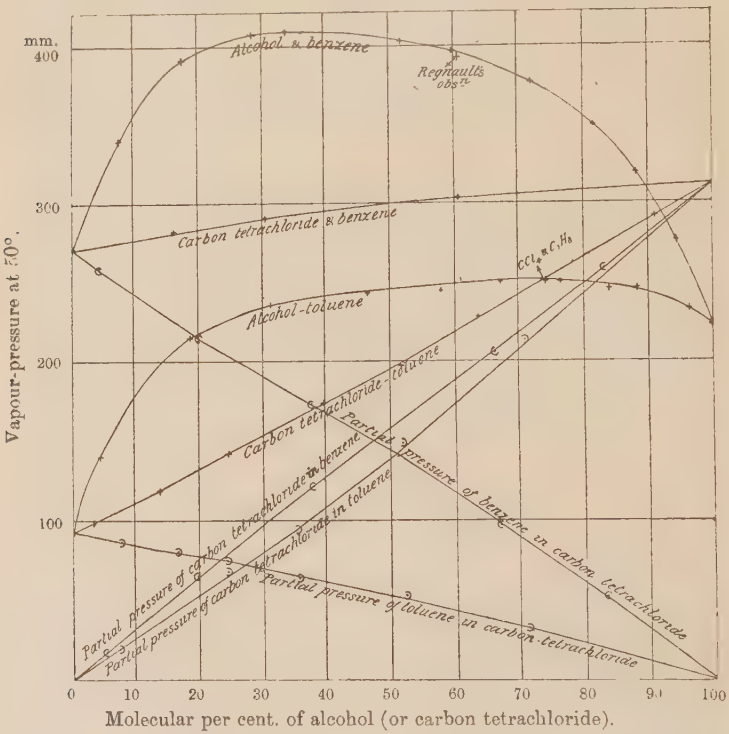
From curve $\frac{d(\log p)}{dt}=0{\cdot}01702$.

The observed temperatures and logarithms of the pressure were, in practice, plotted on a curve (which was approximately a straight line), and the values of $\log p_{50}$ and $d(\log p)/dt$ read off. Here the values of $\log p_{50}$ have been calculated from each observation and the assumed temperature-coefficient, in order to show the degree of concordance obtained. The probable error of $\log p_{50}=0{\cdot}0005$, whence $p_{50}=175{\cdot}3\pm0{\cdot}1$. This reduced to 0° and sea-level in lat. 45° becomes $p_{50}=174{\cdot}8$ mm. The results are shown in the accompanying diagram (fig. 2) and in the tables below. I have only been able to find one earlier observation on the same mixtures: from Regnault's experiments * on mixtures of alcohol and benzene at higher and lower temperatures one finds by interpolation $p_{50}=391$ mm. for a mixture in equal volumes of the liquids, whilst the value from my curve is 393 mm. It is satisfactory to find this confirmation, slight as it is, because the alcohol mixtures are

* Regnault, *Mém. de l'Acad.* xxvi.

the least certain, on account of the inevitable presence of traces of water.

Fig. 2.



The difference between the alcohol curves and those of the normal liquids is most marked; for while mixtures of carbon tetrachloride and benzene, or carbon tetrachloride and toluene, have a vapour-pressure never very different to that which would be obtained by interpolation from the pressures of the pure substances, both mixtures containing alcohol show a maximum—a very flat one. In the case of alcohol and benzene it occurs at about 40 per cent. (molecular) of alcohol, and is 406 mm., being $406/(271+220)=83$ per cent. of the sum of the pressures of the pure substances; while for alcohol and toluene it is at about 74 per cent. of alcohol, and has the value 249 mm., which is $249/(93+220)=80$ per cent. of the sum of the pressures.

With regard to the variation of pressure with temperature, we may, as already remarked, regard $\log p$ as linear in t over a few degrees. But the value of $\partial(\log p)/\partial t$ for the pure substances varies somewhat ; this is partly due to the fact that 50° is a different fraction of the critical temperature in each case ; it diminishes as the " reduced " temperature rises, and consequently has a higher value for toluene at 50° than for benzene or carbon tetrachloride at the same temperature. Alcohol, though its critical point is the lowest of the four, has the highest value of the coefficient—a marked instance of its exceptional behaviour. The numbers are :—

	$\partial(\log p)/\partial t$.
Toluene	0·0189
Benzene	0·0164
Carbon tetrachloride . . .	0·0156
Alcohol	0·0209

The coefficients found for the mixtures are necessarily (on account of the small range of temperature) less reliable than the vapour-pressures themselves : they are in all cases nearly those that would be calculated by interpolation from the coefficients of the pure substances, even when the vapour-pressure curve shows a maximum, *i. e.* they are additive. The differences from the additive values are not greater than the probable errors of experiment. I have included the values of the coefficients in the tables, but lay no stress on them.

Alcohol and Benzene : Vapour-pressures at 50°.

Per cent. alcohol.	Mol. per cent. alcohol.	Saturation-press. p_{50}.	$\frac{d(\log p)}{dt}$.
0	0	270·9	0·0164
4·32	7·11	339·8	
11·11	17·49	389·9	0·0172
18·87	28·28	404·3	179
23·20	33·87	406·4	179
38·81	51·80	400·3	178
46·75	59·80	394·0	188
60·33	72·05	375·3	182
72·37	81·61	345·8	184
81·32	88·07	318·8	175
90·61	94·24	274·3	193
100	100	219·5	209

Alcohol and Toluene.

Per cent. alcohol.	Mol. per cent. alcohol.	Saturation-press. p_{50}.	$\frac{d(\log p)}{dt}$.
0	0	93·0	0·0189
2·14	4·20	141·2	
9·74	17·75	214·8	178
18·26	30·89	233·1	193
29·98	46·13	242·1	193
40·50	57·65	244·2	195
50·15	66·80	249·2	
61·65	76·27	248·2	
71·95	83·69	244·4	
78·60	88·02	243 0	
92·29	95·99	230·9	
100	100	219·5	209

In some of the above cases the temperature-coefficient was not found from experiment, but p_{50} derived from the additive value of it; therefore no number is recorded in the last column.

Carbon Tetrachloride and Benzene.

Per cent. CCl_4.	Mol. per cent. CCl_4.	p_{50}.	$\frac{d(\log p)}{dt}$.
0	0	270·9	0·0164
26·97	15·78	281·0	162
46·32	30·44	290·0	163
75·75	61·31	302·3	161
100	100	310·2	156

These two liquids have so nearly the same vapour-pressure, and the curve between them is so nearly straight, that three observations were considered enough.

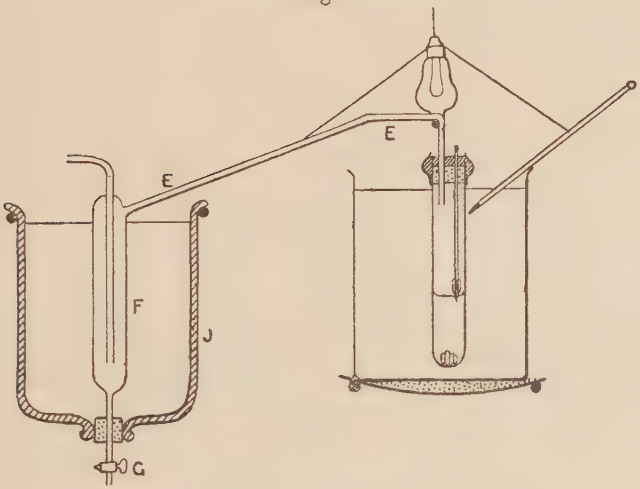
Carbon Tetrachloride and Toluene.

Per cent. CCl_4.	Mol. per cent. CCl_4.	p_{50}.	$\frac{d(\log p)}{dt}$.
0	0	93·0	0·0189
6·52	3·99	99 0	183
20·75	13·51	117·9	174
34·61	24·01	140·8	173
52 40	39·70	174·8	170
63·93	51·42	197·7	167
74·47	63·51	226·5	169
82·64	73·98	248·5	163
94·44	90·99	288·8	156
100	100	310·2	156

Composition of the Vapour over Liquid-Mixtures.

The method used was essentially the same as that described in the previous paper, to distil a little of a mixture and analyse the distillate. The apparatus was, however, arranged so that the distillate could be drawn off by a tap as required; it is shown in fig. 3. It consisted of a boiling-tube, fitted

Fig. 3.



and heated as before; but with the delivery-tube E E bent round to a condenser F. The condenser was provided with a tap G for drawing off the distillate, and a tube which connected to the pressure-gauge, reservoir, and air-pump arranged as before. The condenser was designed so as to permit of the use of a freezing-mixture in the bell-jar J by which it was surrounded; a good many experiments showed, however, that it made no difference to the result whether ice and salt or only cold water was used in J; so water was used for greater convenience. To make the apparatus air-tight, a crucible full of mercury was placed round G, since obviously it was not permissible to lubricate that tap. It is essential to the success of the experiment that no back-condensation should occur, but that the vapour should be collected exactly in the condition it is produced; so the delivery-tube before

the bend must be at least 50°, if the evaporation is taking place at that temperature. This condition was secured by a very simple device: an incandescent lamp with the ordinary conical shade was lowered as close as possible over the water-bath, and a cloth hung round the whole; the electrical heating was then sufficient to keep the top of the apparatus at least as hot as the small flame kept the bottom, and at the same time provided an excellent light to read the thermometers by. About 30 c.c. of mixture was placed in the tube; the temperature of the bath adjusted to between 51° and 52°, and then the pressure lowered till the liquid boiled tranquilly at as near as possible 50°; as the distillation proceeded, the thermometer of course tended to rise; this was corrected by an occasional stroke of the pump as required. Three lots of about 1 c.c. each were usually distilled, and examined separately by the Pulfrich refractometer; from 5 to 10 minutes being required to distil 1 c.c.

In order to interpret the results it was first needful to construct tables of the refractive index of the mixtures used; the refractive index is far from additive in any of the four cases, and cannot be satisfactorily expressed by the parabolic formula used in Part I. for mixtures of benzene with methyl and ethyl acetates; it is therefore worth while to record the results by a short extract from the tables used, which were themselves derived from a smoothed curve based on experiment.

Refractive Indices (at 18° for sodium light).

Per cent. of alcohol (or CCl_4).	Alcohol-benzene.	Alcohol-toluene.	Carbon tetrachloride-benzene.	Carbon tetrachloride-toluene.
0	1·5024	1·4970	1·5024	1·4970
10	1·4869	1·4823	1·5008	1·4956
20	1·4716	1·4680	1·4985	1·4937
30	1·4568	1·4539	1·4958	1·4914
40	1·4425	1·4401	1·4929	1·4889
50	1·4283	1·4265	1·4894	1·4860
60	1·4146	1·4131	1·4853	1·4827
70	1·4011	1·4000	1·4807	1·4787
80	1·3878	1·3873	1·4755	1·4742
90	1·3749	1·3747	1·4692	1·4685
100	1·3622	1·3622	1·4618	1·4618

The numbers recorded in an experiment on the composition of the vapour were (1) the refractometer reading, with the corresponding temperature for the liquid before distillation; (2) that for each of the distillates; (3) that of the residue. From them were calculated the refractive indices, reduced to 18°, and then, from the tables quoted above, the percentage composition of the liquid (mean between the reading before and after), and of the vapour (mean of the three distillates). The results are shown in the following tables :—

Composition of Vapours.

A, B = molecular weights of the two components.
z = fractional composition of liquid.
y = " " vapour.
ζ = molecular fractional composition of liquid.
η = " " " vapour.
q = ratio of masses of substances A and B present in liquid.
t = " " " " vapour.
χ = ratio of number of molecules of A and B present in liquid.
s = " " " " vapour.
p = total vapour-pressure.
p_A = partial pressure due to A.
p_B = " " B.

Alcohol–Benzene.

A (Alcohol) = 45·70. B (Benzene) = 77·46.

z.	y.	ζ.	η.	log q.	log t.	p (from curve).	p_A (from p & η).	p_B (from p & η).
0·05·4	0·18·7	0·08·8	0·28·1	$\bar{2}$·7566	$\bar{1}$·3617	350·4	98·5	251·9
0·07·5	0·21·9	0·12·1	0·32·2	$\bar{2}$·9090	$\bar{1}$·4478	369·0	118·8	250·2
0·13·9	0·24·7	0·21·5	0·35·7	$\bar{1}$·2079	$\bar{1}$·5159	397·0	141·6	255·3
0·24·5	0·27·5	0·35·5	0·39·1	$\bar{1}$·5112	$\bar{1}$·5790	406·0	158·7	247·2
0·32·0	0·29·7	0·44·4	0·41·7	$\bar{1}$·6727	$\bar{1}$·6258	404·4	168·6	235·8
0·43·0	0·32·6	0·56·1	0·45·1	$\bar{1}$·8776	$\bar{1}$·6846	397·6	179·3	218·3
0·58·0	0·38·3	0·69·7	0·51·3	0·1326	$\bar{1}$·7929	378·4	194·1	184·3
0·82·1	0·54·3	0·88·6	0·66·8	0·6615	0·0748	315·0	210·4	104·6

Alcohol–Toluene.

A=45·70. B (Toluene) =91·37.

z.	y.	ζ.	η.	log q.	log t.	p.	p_A.	p_B.
0·07·4	0·41·9	0·13·8	0·59·1	$\bar{2}$·9025	$\bar{1}$·8581	119·5	117·9	81·6
·20·0	·49·1	·33·4	·65·9	$\bar{1}$·3989	$\bar{1}$·9843	235·0	154·8	80·2
·28·0	·51·6	·43·7	·68·1	$\bar{1}$·5898	0·0278	241·0	164·1	76·9
·36·3	·54·4	·53·3	·70·5	$\bar{1}$·7558	0·0766	245·0	172·8	72·2
·46·4	·55·4	·63·4	·71·3	$\bar{1}$·9374	0·0941	247·0	176·1	70·9
·58·2	·58·6	·73·6	·73·9	0·1436	0·1508	249·0	184 0	65·0
·67·5	·62·1	·80·6	·76·6	0·3174	0·2146	246·5	188·8	57·7
·79·3	·72·3	·88·5	·83·9	0·5833	0·4163	241·5	202·6	38·9
·89·8	·82·1	·94·6	·90·2	0·9447	0·6615	233·5	210·6	22·9

Carbon tetrachloride–Benzene.

A (Carbon tetrachloride) =152·70. B=77·46.

z.	y.	ζ.	η.	log q.	log t.	p.	p_A.	p_B.
0·18·1	·10·1	·04·3	·05·4	$\bar{2}$·9450	$\bar{1}$·0508	273·6	14·7	258·9
·32·6	·37·4	·19·7	·23·3	$\bar{1}$·6846	$\bar{1}$·7763	280·0	65·2	214·8
·54·3	·58·0	·37·6	·41·2	0·0748	0·1402	294·0	121·1	172·9
·79·3	·80·5	·66·0	·67·7	0·5833	0·6157	303·4	205·3	98·1
·90·6	·90·9	·83·0	·83·5	0·9840	0·9995	306·5	255·8	50·7

Carbon tetrachloride–Toluene.

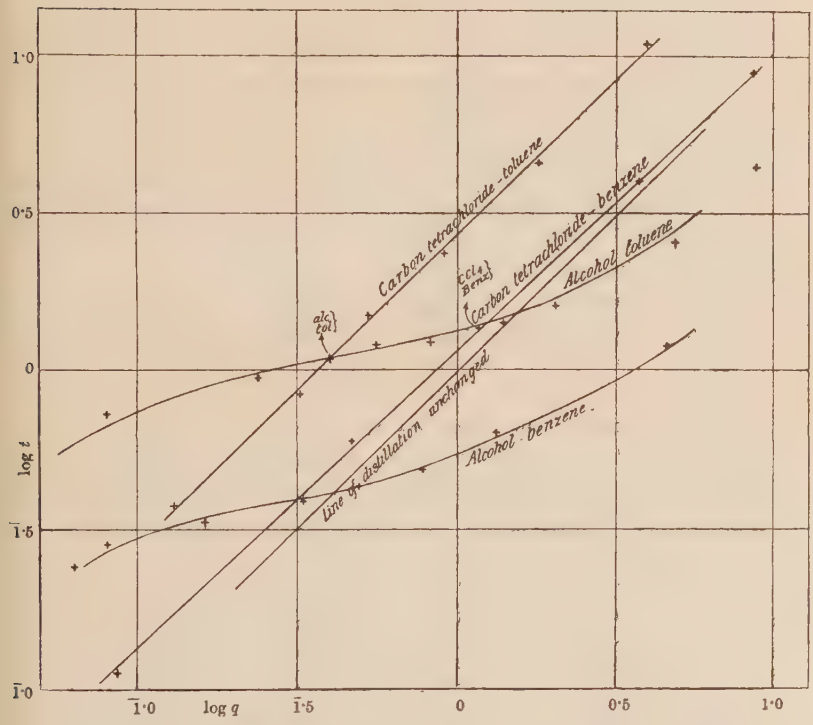
A=152·70. B=91·37.

z.	y.	ζ.	η.	log q.	log t.	p.	p_A.	p_B.
0·11·7	0·27·0	0·07·3	0·18·1	$\bar{1}$·1222	$\bar{1}$·5681	105·4	19 0	86·4
·24 6	·46·7	·16·3	·34·4	$\bar{1}$·5136	$\bar{1}$·9426	122·8	42·2	80 6
·34·5	·59·7	·24·0	·47·0	$\bar{1}$·7216	0·1706	140·0	65·8	74·2
·47·4	·71·2	·35·0	·59·7	$\bar{1}$·9548	0·3930	158·0	94·3	63·7
·64·1	·82·8	·51·7	·74·2	0·2519	0·6825	198·9	147·6	51·3
·80·2	·91·9	·70·8	·87·2	0·6076	1·0550	242·0	211·0	31·0

It is difficult to say what is the degree of accuracy of these results, but certainly less than the accuracy of the vapour-pressure measurements, chiefly because some of the condensed vapour clings to the condensing-tube, and it is not certain that what is collected quite fairly represents the whole.

These results are also expressed by the curves on fig. 4. Considering first, then, two pairs of normal liquids, we see that the relation $\log t = \log k + r \log q$, where k and r are

Fig. 4.



constants, satisfactorily represents the phenomena. That is the relation proposed in the previous paper for mixtures of benzene with methyl and ethyl acetates, and in all these cases it appears to be correct within the limits of error of experiment. But Margules has quite rightly pointed out (*loc. cit.* p. 1266) that when $r<1$ (as it usually is) the equation leads to infinite values of $\partial p/\partial \zeta$ when $\zeta=0$ or $\zeta=1$, *i. e.*, the

vapour-pressure curve would start and end vertically. That deduction I had overlooked, and it is obvious from the curves on fig. 2 that the vapour-pressure curve does not behave in that way. The relation given cannot therefore hold throughout; still the experiments show it to be a remarkably good empirical rule for mixtures which do not contain a very small proportion of either component, extending in the case of carbon tetrachloride-benzene mixtures from 8 to 91 per cent.

The empirical linear relations are then :—

For carbon tetrachloride–benzene ... $\log t = 0{\cdot}065 + 0{\cdot}947 \log q$,
„ carbon tetrachloride–toluene ... $\log t = 0{\cdot}440 + 1{\cdot}0 \log q$.

These relations are intended as a solution of the differential equation (11) in the foregoing paper. Margules proposes a more complicated solution, which satisfies the condition that for dilute solutions the lowering of the vapour-pressure of the other component (the solvent) should be normal, *i. e.* according to van 't Hoff and Raoult's rule. It is doubtless possible to represent the experimental results that way by using a sufficient number of arbitrary constants; but I have not thought it worth while to do so, as it is complicated, and seems to go rather beyond what the experimental data justify.

With regard to the other mixtures, alcohol with benzene and toluene, the diagram shows the relations between $\log t$ and $\log q$ to be very far from linear. I have not attempted to find an equation to these curves, and will only make one or two remarks as to their character. They are not drawn directly from the experimental results on the composition of the distillate, because when the percentage of alcohol is very great or very small the observations lose considerably in accuracy; but on calculating the partial pressures, from the observed composition of the vapour, and the total pressure, smoothed curves could be drawn, the reliability of which is enhanced by the fact that their end points (vapour-pressure of the pure substances) are fixed. From these smoothed curves of partial pressure the compositions were calculated afresh, and it is the numbers so obtained that are represented by the curves on fig. 4. The crosses on that figure stand for the immediate results of observation, and it will be

noticed that they are regular enough in the middle parts of the curves but not at the ends.

At the maxima of vapour-pressure the composition of the liquid and the vapour must be the same. That is shown, consequently, by the intersection of the curves with the line $\log q = \log t$. The points of intersection form the most reliable measure of the position of the maxima. They give

	$\log q$.	z.	ζ.
Alcohol-benzene.....	1·605	0·28·7	0·40·6
Alcohol-toluene.....	0·170	0·59·7	0·74·7

It will be seen that these results agree well with the maxima shown in the vapour-pressure curves (fig. 2).

The differential equation referred to above is

$$\left(\frac{Bq}{Bq+A} - \frac{s}{s+1}\right)\frac{1}{s}\frac{\partial s}{\partial q} + \frac{1}{p}\frac{\partial p}{\partial q} = 0,$$

or, as it may be written,

$$(\eta - \zeta)\frac{\partial \log s}{\partial \log q} = \frac{\zeta(1-\zeta)}{p}\frac{\partial p}{\partial \zeta}.$$

For some purposes it is more convenient to express it in terms of the partial pressures, when it assumes the more symmetrical form adopted by Margules (*loc. cit.*) :

$$\zeta\frac{\partial \log p_A}{\partial \zeta} + (1-\zeta)\frac{\partial \log p_B}{\partial \zeta} = 0.$$

Since, however, the relation between q and s (or between ζ and η) has not so far been expressed successfully by an equation, it is not possible to integrate the differential equation in order to compare it with experiment. This is true even for the mixtures of benzene and toluene with carbon tetrachloride, since the empirical equations, quoted above, do not extend through the whole range of q, and therefore the arbitrary constant in the integral equation cannot be determined. We are reduced therefore to the very rough process of comparing the differential equation itself with

experiment, by measuring the slope of the vapour-pressure curves at various points. The relation $\log t = \log k + r \log q$ or $\log s = \log Bk/A + r \log q$ gives $\partial(\log s)/\partial(\log q) = r$, and hence

$$\eta - \zeta = \frac{\zeta(1-\zeta)}{rp}\frac{\partial p}{\partial \zeta}.$$

The agreement shown between the two sides of this equation is very rough. Thus :

Carbon tetrachloride and Toluene.

ζ.	0·073	0·163	0·240	0·350	0·517	0·708
$\eta - \zeta$.....	0·105	0·181	0·230	0·247	0·225	0·164
$\frac{\zeta(1-\zeta)}{rp}\frac{\partial p}{\partial \zeta}$	0·123	0·233	0·276	0·311	0·278	0·198

Carbon tetrachloride and Benzene.

ζ.	0·043	0·197	0·376	0·660	0·830
$\eta - \zeta$.....	0·011	0·036	0·036	0 017	0·005
$\frac{\zeta(1-\zeta)}{rp}\frac{\partial p}{\partial \zeta}$	0·010	0·037	0·040	0·017	0·009

Whether the disagreement is due to the difficulty of treating the differential equation directly, to errors of experiment, or to the approximations in the theory, I am at present unable to say.

In the case of the alcohol mixtures Margules's form of the equation was adopted, and the differential coefficients measured from the smoothed curves. The results following show, it will be seen, the kind of agreement that might be expected in the middle of the range, but in the case of alcohol and benzene disagree totally at the extreme percentages.

Alcohol and Benzene.

ζ.	0·10	0·30	0·50	0·70	0·90
$\frac{\zeta}{p_A}\frac{\partial p_A}{\partial \zeta}$	0·67	0·19	0·27	0·34	0·40
$\frac{1-\zeta}{p_B}\frac{\partial p_B}{\partial \zeta}$	0·31	0·16	0·34	0·46	0·82

Alcohol and Toluene.

ζ.	0·10	0·30	0·50	0·70	0·90
$\frac{\zeta}{p_A}\frac{\partial p_A}{\partial \zeta}$	0·61	0·23	0·21	0·27	0·62
$\frac{1-\zeta}{p_B}\frac{\partial p_B}{\partial \zeta}$	0·54	0·22	0·25	0·27	0·70

It has already been mentioned that Linebarger states the conclusion that the partial pressure of benzene and toluene in mixtures is simply proportional to the molecular percentage present; this conclusion appears to be only roughly true. According to the results expressed by the curves on fig. 2 no such simple relation holds; the partial pressures of these hydrocarbons being as far from linear as that of the carbon tetrachloride with which they are mixed.

Linebarger proceeds to apply his conclusion to determine the molecular weight of acetic acid, in mixtures of the acid with benzene and toluene.

For in any mixture the partial pressure of the hydrocarbon p_B divided by the pressure of the pure liquid π_B gives, according to this rule, the true molecular fraction of hydrocarbon present, say $1-\zeta'$, whilst $1-\zeta$ is that calculated from the formulæ C_6H_6 (or C_7H_8) and CH_3COOH, or

$$1-\zeta=p_B/\pi_B,$$

whence

$$\chi'=\zeta'/(1-\zeta')=(\pi_B-p_B)/p_B,$$

where χ' is the true ratio of the number of molecules of acetic

acid to molecules of hydrocarbon. If this be divided into the ratio χ calculated from the formulæ, the quotient will give the degree of aggregation of the molecules of acetic acid in the solution. In this way Linebarger found molecular weights which steadily increased with the concentration of the acid, and by extrapolation found 240 for the molecular weight of the pure liquid at 35°. My results for alcohol do not, however, at all confirm the accuracy of the method. On drawing smooth curves of partial pressure from the observations given in the tables above, and treating them in the way just indicated, we get

Molecules of Alcohol.	Aggregation of the Alcohol Molecules in	
	Benzene.	Toluene.
10	6·0	2·0
30	5·7	2·6
50	5·3	4·2
70	5·0	5·7
90	4·9	6·0

It will be seen that the two series disagree altogether, and that neither leads to any reasonable value for the aggregation of pure alcohol. I can only conclude that the partial pressure of the hydrocarbon vapour is not necessarily linear in mixtures, and that therefore the rule proposed by Linebarger for determining the molecular weight of liquids is not correct.

Further theoretical conclusions would not, I think, be justified at the present time on account of the small amount of material accumulated by experiment.

Davy-Faraday Laboratory of the Royal Institution,
London, December 1897.

XVII. *On some Improvements in the Roberts-Austen Recording Pyrometer, with Notes on Thermo-Electric Pyrometry.* *By* Alfred Stansfield, *B.Sc., A.R.S.M., Royal College of Science, London* *.

The method of recording pyrometry which involves the use of the thermo-junction has, as is well known, been devised by Prof. Roberts-Austen. He suggested that the author should undertake the investigations described in the following paper, and the experiments were conducted in the laboratory of the Royal Mint.

Introduction.

An excellent summary of the early work on pyrometry is given by Barus in his elaborate paper "On the Thermo-Electric Measurement of High Temperatures" †, published in 1889, so that it will only be necessary here to consider briefly the present condition of high-temperature measurement.

The ultimate standard of temperature is afforded by the air-thermometer; nearly all other instruments are referred to this either directly or indirectly. Apart from the air-thermometer, which is not well adapted for ordinary use, there are two systems of pyrometry suitable for exact measurements of temperature. One of these, which depends on the change produced in the electrical resistance of a coil of platinum wire by a change of temperature, was adapted to industrial use by Sir W. Siemens, and has been greatly improved and widely utilized by Callendar and Griffiths, and by Heycock and Neville. The other system utilizes the electromotive force produced in a circuit of two or more metals when one junction is heated; its early use is associated with the names of Becquerel (1826) and Tait (1873), while more recently it has been employed by H. Le Chatelier, Barus, and Roberts-Austen. The thermo-couples in general use consist of platinum and platinum alloyed with about 10 per cent. of either

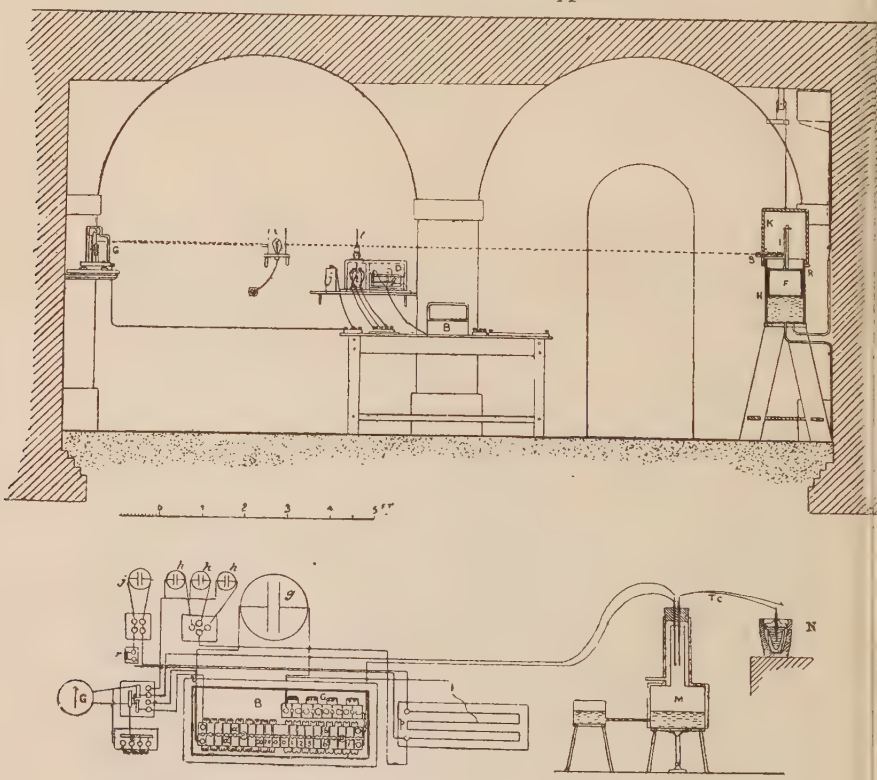
* Read March 25, 1898.

† Barus, Bull. U.S. Geol. Survey, no. 54. Washington, 1889.

iridium or rhodium, the former alloy having been first used by Tait, the latter by Le Chatelier.

The platinum-resistance pyrometer, or platinum thermometer, as it is often called, appears to be capable of affording more accurate results than the thermo-couple especially up to about 600°. This mainly arises from the fact that the measurement of a resistance can be made with a greater

Fig. 1.—Arrangement of the Apparatus.



degree of accuracy than the measurement of the extremely small E.M.F. produced by the thermo-couple. Moreover, the whole of the coil of wire in the resistance-pyrometer is in the region the temperature of which is to be measured; hence the indications do not depend, as do those of the thermo-couple, on the absolute uniformity of composition of the

platinum wire employed. At higher temperatures, however, the resistance-pyrometer is more seriously affected than is the thermo-couple by the difficulty of securing good insulation, and by the difficulty of obtaining uniformity and constancy of the temperature to be measured ; although the difficulty of insulation has been practically overcome by Callendar's use of mica supports. The resistance-pyrometer, therefore, although it may be capable of affording more accurate results than the thermo-couple, is much less convenient for use at high temperatures. The latter, consisting of two thin wires which may be inserted into a small protecting tube of fireclay or porcelain, can be used to measure the temperatures of very small enclosures, and its relatively small lag renders it suitable for obtaining autographic temperature records.

The present research was begun with the object of modifying the Roberts-Austen pyrometer, in order that it might be used to obtain large-scale temperature records having as high a degree of accuracy as possible. It was intended to study the degree of constancy which could be obtained in the indications of the thermo-couple ; to make observations of the melting-points of several metals by means of a thermo-couple; and to calibrate the couple by means of the porcelain air-thermometer. The present paper describes the Roberts-Austen recording pyrometer, and the changes which have been made in view of the above objects, together with the precautions which should be observed in order to avoid errors in the use of the thermo-couple. It also contains an account of the results that have been obtained, and the bearing of these results on the theory of the thermo-couple.

The Roberts-Austen Recording Pyrometer.

The thermo-couple has been used in pyrometry in two ways :—(1) by measuring the E.M.F. of the heated couple by a null method, and (2) by observing the deflexion of a galvanometer to which it is connected.

In the recording pyrometer of Roberts-Austen * the latter method is adopted ; the thermo-couple is connected to a galvanometer the deflexion of which is recorded on a moving

* Proc. Roy. Soc. 1891, vol. xlix. p. 347.

photographic plate. The resistance of the galvanometer is sufficiently large to render changes in the resistance of the thermo-couple unimportant, and as the instrument is calibrated directly by observations of known melting or boiling temperatures, it is not necessary to assume any definite relation between the current and the deflexion of the galvanometer. This form of pyrometer is extremely convenient and useful, and the photographic records taken by its aid of the cooling of an alloy of two or more metals frequently show small evolutions of heat which could not easily be detected by direct observation. These evolutions of heat, even when extremely small, are of great importance in studying the constitution of alloys. A few records obtained in this way are given in fig. 2, which is a reproduction of a photographic plate on which several records have been taken in succession. Three of these, B, C, and D, represent the cooling of alloys of copper and tin * containing respectively 50, 55, and 45 per cent. of copper.

In these alloys, most of the halts in the cooling-curves represent the solidification of eutectic alloys. The curves A and Z represent the cooling and solidification of aluminium and of zinc, and the known melting-points of these metals have enabled a temperature-scale to be constructed for use with the other records.

In this form of recording pyrometer the accuracy of the record depends on the constancy of the galvanometer, as well as on that of the thermo-couple itself, so that another element of uncertainty is introduced into the results. In addition to this, the deflexion of the galvanometer for the highest temperature to be measured must be limited to the width of the photographic plate employed if the zero of the scale is to be recorded on the same plate. In measuring high temperatures, therefore, the sensibility of the galvanometer must be small, and consequently the record will be on a small scale.

For general use, these disadvantages are quite outweighed by the fact that the whole record of the cooling of an alloy can be obtained in one curve, but for special purposes, such

* Alloys Research Committee, 3rd and 4th Reports: Proc. Inst. Mech. Eng. 1895, p. 269, and 1897, p. 67.

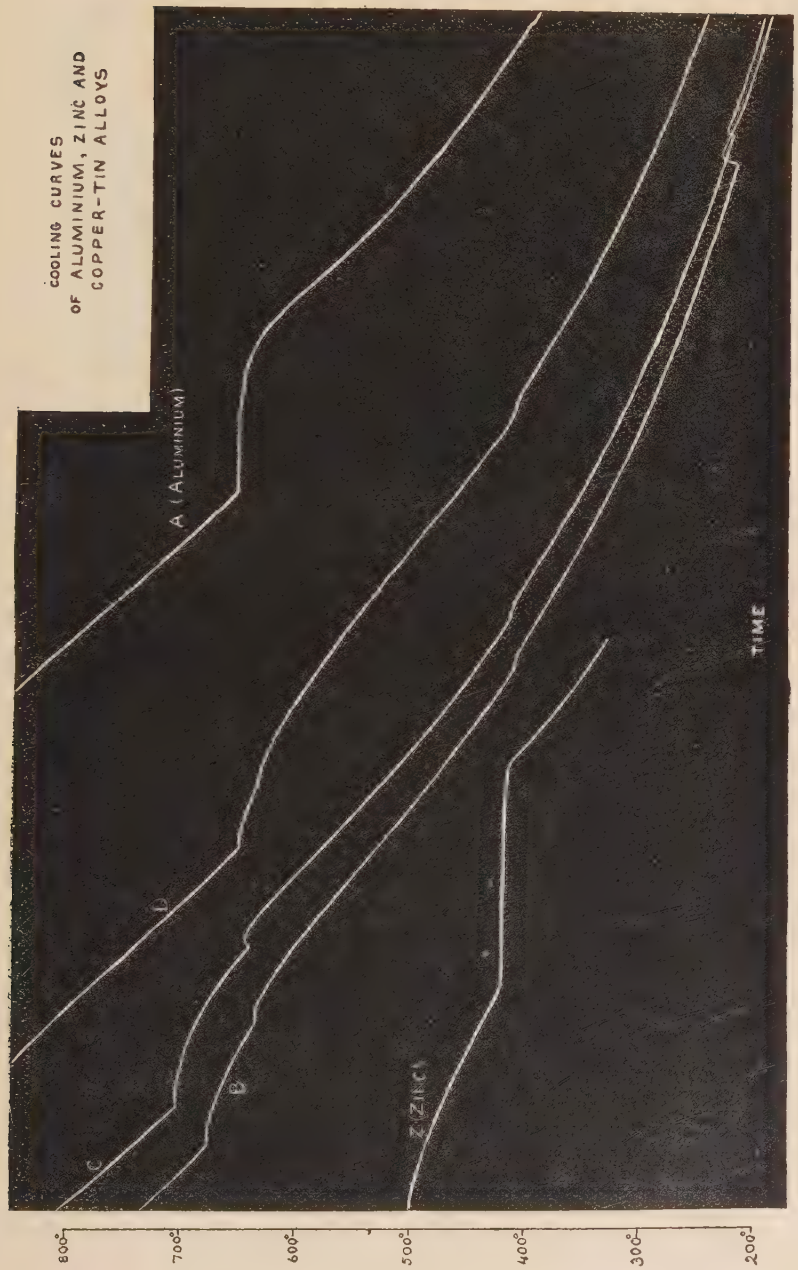

COOLING CURVES
OF ALUMINIUM, ZINC AND
COPPER-TIN ALLOYS
A (ALUMINIUM)
D
C
B
ZINC
TIME
800°
700°
600°
500°
400°
300°
200°

Fig. 2.

as obtaining accurate measurements of melting- or boiling-points and investigating the exact shape of the "cooling-curve" of a metal or alloy at the temperature of freezing or of other molecular change, a modification of the method becomes necessary. In order to obtain large-scale records of these changes, Roberts-Austen employed a more sensitive galvanometer, its zero position being adjusted by trial until the spot of light from its mirror fell upon the photographic plate when the thermo-couple was at the particular temperature at which a record of the cooling was desired. The zero position of the enlarged record was often many feet from the plate, and, as the galvanometer had to be deflected through a large angle, its indications became untrustworthy, and it was necessary to employ simultaneously a second ordinary galvanometer to give a measurement of the temperature at which the enlarged record was taken. The two galvanometers were connected in parallel and arranged one behind the other *, so that they both recorded their deflexion on the same moving plate. A record taken in this way is given in fig. 3.

It represents the melting and solidification of an ounce of pure gold; the thermo-junction was immersed in the molten gold without any covering, and had therefore scarcely any lag other than that produced by the galvanometer itself. The curves A and B were produced by the less sensitive galvanometer, and the scale on the left has been obtained by measuring the ordinates of these curves from the datum on the original plate. The scale for the more sensitive records C and D is 22 times as great as the other, and it is obtained from the latter by comparing the slopes of the two records.

The curves A and C represent the melting, and the curves B and D the solidification of the gold. The irregularities in the melting record probably indicate the fall of pieces of still solid gold into the molten mass. On account of this irregularity *cooling*-curves are usually employed, instead of heating-curves, in investigating the constitution of metals and alloys. The curve representing the melting of the gold is nearly 1° higher on the scale than the curve representing

* Roberts-Austen, Proc. Inst. Mech. Eng. 1895, p. 243, and fig. 4, pl. xxxvii.

its solidification, and it would appear that the true melting-point of a pure metal is indicated most nearly by that part of the cooling record where the curve first becomes approximately horizontal.

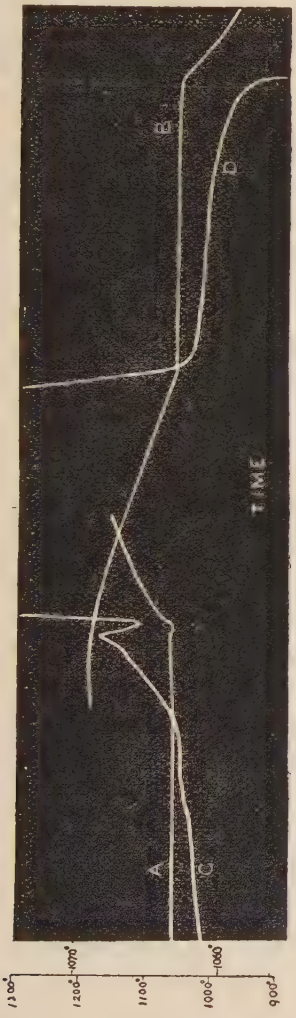


Fig. 3.—Melting and Solidification of Gold.

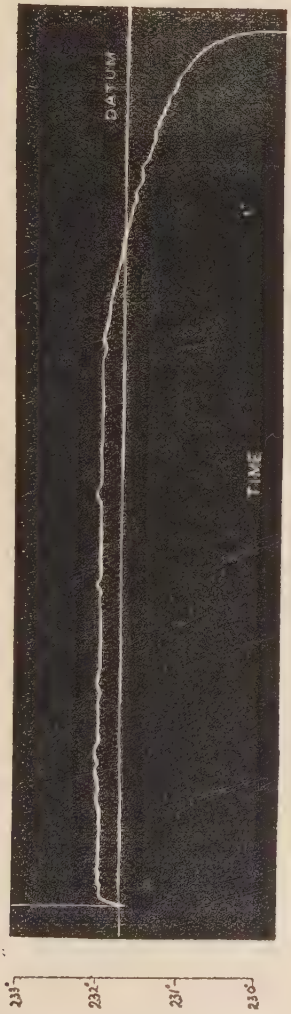


Fig. 4.—A record of the Solidification of Tin.

The author considered two methods for avoiding the large deflexion of the sensitive galvanometer and thus rendering its indications more trustworthy :—(1) that of maintaining the

"cold junction" of the thermo-couple at some high temperature, such as the boiling-point of sulphur, thus measuring *differences* between two high temperatures; and (2) that of balancing the greater part of the E.M.F. of the thermo-couple by a potentiometer method, recording the small remainder by means of a sensitive galvanometer on a moving photographic plate. The latter method was adopted as being much more convenient and generally applicable.

The New Form of Recording Pyrometer.

The Potentiometer.—As the ordinary stretched-wire form of potentiometer, with sliding contacts, would have been unsuitable for the purpose, a special instrument was constructed with plug contacts by means of which any number, from 1 to 99, of equal units of E.M.F. could be inserted in the galvanometer circuit.

The potentiometer, which is shown at B in fig. 1, consists of four sets of resistance-coils; a, a' are sets of nine resistances of 2 ohms each, and b, b' of 0·2 ohm each, while c is a set of 1000 ohm coils which are placed in the battery circuit. The coils a, b are always in circuit with the galvanometer G and thermo-couple Tc. The battery current flows through the set of resistances c and through parts of the resistances in a' and b', and the circuit is completed by the resistances in a and b which lie between the plugs P, P′.

The terminals of the resistances in a, a' are numbered 0, 10, 20, &c. to the left, and in b, b' they are numbered 0, 1, 2, &c. to the right, so that the readings opposite the two plugs give the number of units of E.M.F. in the galvanometer circuit. This arrangement, which involves the use of a larger number of resistance-coils than is usual in potentiometers, was adopted in order that the resistances of the galvanometer circuit and of the battery circuit might remain the same for all positions of the plugs P, P′. The relative resistances of the coils a, b were carefully measured, and a table was calculated giving their values in terms of the mean unit. Changes in the temperature of the potentiometer should not affect the E.M.F. included in the galvanometer circuit, provided that all the coils are at the same temperature. As it was feared that the high-resistance coils c would become

heated by the passage of the current to a greater degree than the low-resistance coils, the box B was provided with a zinc lining and the coils were so arranged they might all be immersed in paraffin, which could be kept in circulation by means of a stirrer. Experiments on the relative resistances of the high and low resistance-coils failed to show that any change was produced by the passage of the extremely small potentiometer current, and the use of the paraffin was, therefore, not resorted to.

The current flowing through the potentiometer is maintained by means of a Clark cell *g* with large electrodes, each electrode being 20 sq. in. in area. The Clark cell is placed in series with a resistance of 2500 ohms. Under these circumstances, as has been shown by S. Skinner *, the current produced is remarkably constant, and there is also this advantage in the use of the Clark cell, that its E.M.F. when producing a current can be easily compared with that of standard Clark cells of the usual form. The use of Clark cells for standards has necessitated a large temperature correction; but as the cells were placed in a thick wooden box D, and their temperature only rose 2 or 3 tenths of a degree during the day, the rate of change of temperature was sufficiently slow to render this correction fairly legitimate. The E.M.F. of the large Clark cell *g* is compared with that of the standards *h*, *h*, *h* by means of the auxiliary potentiometer *p*, which is connected in series with a dry cell *j* and resistance *r* (the necessary connexions can easily be traced on the figure). This potentiometer measures the small difference between the E.M.F. across the terminals of the main potentiometer and the E.M.F. of each of the standards *h*, *h*, *h*. By an arrangement which is not shown in the figure, the E.M.F. across the terminals of *p* is obtained in potentiometer units, and so the readings on *p* can be converted into a correction to be applied to the readings in B.

The Galvanometer.—The sensitiveness of the pyrometer depends on the sensitiveness of the galvanometer and its constancy of zero. The difficulty of obtaining a sufficiently sensitive galvanometer was increased by the necessity for introducing the resistance (20 ohms) of the potentiometer.

* Phil. Mag. Sept. 1894, p. 271; Proc. Phys. Soc., Feb. 1895.

In experimenting with sensitive suspended-coil galvanometers, great difficulty was experienced in finding one which would return to its zero position after being deflected. This difficulty was at first attributed to the viscosity of the phosphor-bronze strip used to suspend the coil, but it was found that the coil would vibrate when suspended by a silk fibre between the poles of a permanent magnet, a fact which showed that the coil itself was magnetic and that the change in zero was largely due to a change in the direction of the induced magnetism.

A galvanometer, designed to be as free as possible from this defect, was made by Dr. Muirhead; the coil was shuttle-shaped, instead of being hollow as in the ordinary form, and was suspended by a strip at the top; the connexion at the bottom was made by a fine spiral. This instrument returned to its zero-reading fairly well after small deflexions, but for large deflexions it was less reliable than the older form; it was, moreover, much more susceptible to vibrations, a fact which greatly reduced its usefulness, as it was impossible to obtain a reasonably smooth photographic record of its readings. All the records shown were obtained by the open-coil galvanometer.

The Recording Apparatus, shown at H in fig. 1, consists of a photographic plate I, mounted on a float F, which slowly rises as water is admitted into the cylinder H. Vertical glass rods at the sides of the cylinder act as guides to prevent rotation or lateral movement of the float. The whole is covered by a hood K, which can readily be raised by means of the pulley and counterpoise. It is supported by a "hole, slot, and plane," on the fixed board R, with which it makes a light-tight joint. A slit S, which forms part of the hood, permits only horizontal beams of light to reach the plate; consequently it is possible to have the room fully illuminated without danger of fogging the plate. The source of light L is an ordinary glow-lamp enclosed in a wooden box. A brass tube, with a rectangular diaphragm at the end nearest the lamp, cuts off all the light except that from a selected piece of its vertical filament. Light from this filament is reflected by means of a plane mirror on the galvanometer, and focussed on the photographic plate by a lens in front of the galvanometer.

This method of focussing the light, which was suggested by Prof. Boys, enabled a fine line of light to be obtained for recording, notwithstanding the fact that in order to obtain records on a sufficiently large scale the galvanometer was placed 16 feet from the photographic plate. This line of light passing through a horizontal slit immediately in front of the plate produces a square spot on the plate. The glow-lamp is very convenient for ordinary work, but the light is insufficient for taking rapid records, and it becomes necessary, in taking such records, to employ a lime-light.

The Thermo-Couple consists of two wires, one of platinum and the other of platinum alloyed with 10 per cent. of rhodium or iridium. The wires are fused together at the end and inserted for protection into a thin fire-clay tube. Each wire was insulated from the other, except at the junction, by means of a slip of mica. The free ends of the wires were soldered to the leads from the recorder, and these "cold junctions" were kept in ice, or in cold water the temperature of which was noted. More recently an hypsometer has been used, as shown at M in the figure, thus keeping the "cold junctions" at the temperature of boiling water, which has the advantage of being more constant than the temperature at which cold water can readily be maintained; the hypsometer also requires less attention than a "cold junction" involving the use of ice. The interval of temperature to be measured is also reduced, and errors in the E.M.F. are decreased in the same proportion.

Method of taking a Record.—The photographic plate is first placed on the float F, using a red light in the room, and a cover is placed over the slit to keep out the spot of light. The galvanometer is short-circuited by the shunt-box *e*, and the thermo-couple, in its clay tube, is placed in the metal or alloy, N, which has been melted in a gas-furnace. The flame is then either turned out or reduced in size, so that the furnace may cool slowly. The plugs P, P′ are placed in position in the potentiometer, and the shunts are gradually removed from the galvanometer as the balance is obtained. When the galvanometer has its full sensibility, the spot of light travels almost across the photographic plate every time that the position of P′ is changed in order to maintain the balance as

the furnace cools down. This change is effected by placing another plug immediately to the left of P′ before the latter is removed; when it becomes necessary to move the plug P the galvanometer is short-circuited. When the metal has cooled to the point at which a record is desired, the water-clock is started, and the cover removed from the slit to admit the spot of light from the galvanometer to the photographic plate. As the movement of the spot of light can be seen on a scale immediately above the slit, it is easy to follow the progress of the experiment without interfering with the record. As the metal cools down and the photographic plate rises, successive tracks of the spot of light may be obtained on the plate, each track corresponding to a particular position of the plugs. When the cooling has been carried sufficiently far, the galvanometer is disconnected from the thermo-couple and short-circuited by a contact-piece at *f*, and a datum line is run on the plate to indicate the zero position of the galvanometer.

It is also necessary

(1) To take the barometer-reading to determine the temperature of the "cold junction."

(2) To obtain the value of the potentiometer E.M.F. in terms of the standard Clark cell by means of the auxiliary potentiometer.

(3) To calibrate the auxiliary potentiometer.

(4) To read the temperature of the Clark cells by means of the thermometer *t*.

(5) To determine the relation between the deflexion of the galvanometer and the potentiometer-unit.

Fig. 4 is a reproduction of a record, obtained in this way, of the solidification or freezing of tin. The metal was contained in a small clay crucible inside a cast-iron chamber heated externally, so that very slow and regular cooling could be obtained.

The metal was stirred during the solidification in order to promote uniformity of temperature and to prevent surfusion. It will be noticed that the temperature remained practically constant until the greater part of the metal had solidified.

Records have also been taken on the same scale of the

freezing of metals having much higher melting-points, such as gold or copper; but the difficulty of obtaining a satisfactory curve is greatly increased.

Discussion of the Observations.

In order to discuss the theoretical bearing of the readings of the thermo-couples it is necessary to know the temperatures to which they correspond. Although measurements of high temperatures have been made with the air-thermometer, they are not given in the present research. The results of different experimenters have nevertheless determined the melting-temperatures of silver, gold, and copper, with an error of probably less than 10°; the relative temperatures being known much more accurately. For the purposes of the mathematical treatment of the results, the melting-temperature determinations of Heycock and Neville may be taken. Although their measurements of the melting-temperatures of silver, gold, and copper depend on a somewhat severe extrapolation of a formula, their determinations of the *relative* melting-points of these metals are probably very accurate, and their results agree fairly well with those obtained by the direct air-thermometer observations of Holborn and Wien.

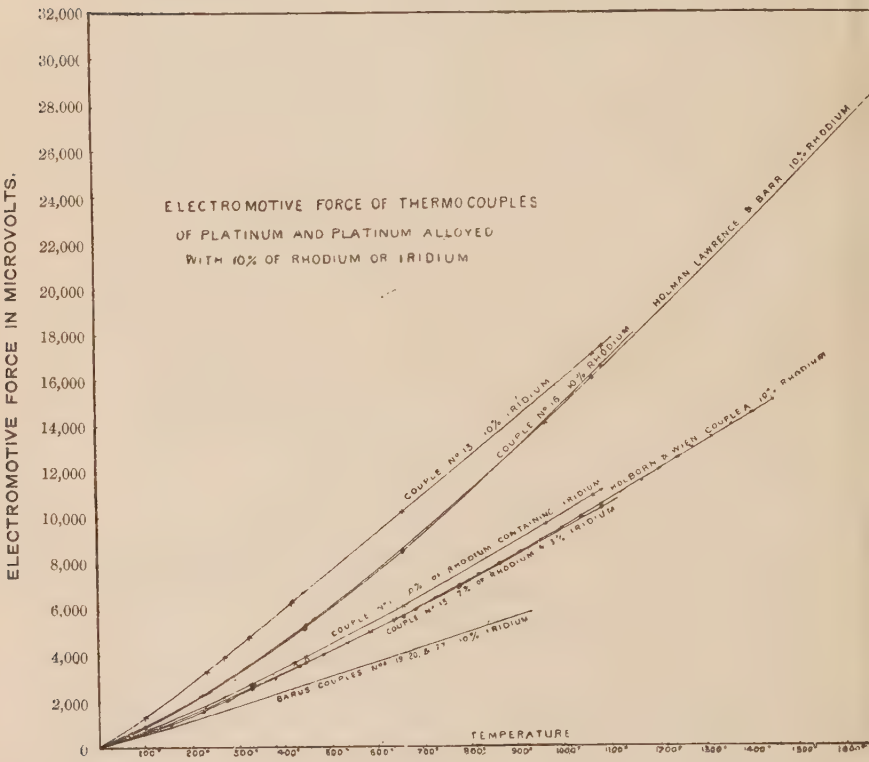
In fig. 5 the results obtained by the author for different thermo-couples are plotted in the usual way; the co-ordinates representing the temperature of the hot junction (t), and the observed E.M.F. of the couple (E), the cold junction being supposed to be at 0° C.

Curves representing some of the observations of Barus, Holborn and Wien, and Holman, Lawrence, and Barr have also been plotted. An examination of figure 5 shows that very serious discrepancies exist between the indications of couples having the same nominal composition. Thus the thermo-electric power of the 10 per cent. rhodium thermo-couple of Holborn and Wien was only about 3/5 of that obtained by Holman, Lawrence, and Barr; and the 10 per cent. iridium couple employed by the author had about $2\frac{1}{2}$ times the thermo-electric power of a 10 per cent. iridium alloy examined by Barus. Through the kindness of Colonel Edward Matthey, the author has been able to investigate the causes of these discrepancies, which are too large to be

attributed merely to the accidental differences in the percentage of rhodium or iridium in the platinum alloy used.

Two 10 per cent. rhodium alloys were prepared and drawn into wires from which the thermo-couples, Nos. 15 and 16, were made up with a return wire of pure platinum common to

Fig. 5.



both. The rhodium used in the composition of No. 15 was part of a commercial sample and was found to contain no less than 30 per cent. of iridium, so that the alloy consisted of 90 per cent. platinum, 7 per cent. rhodium, and 3 per cent. iridium; No. 16, on the other hand, contained 10 per cent. of pure rhodium. Observations made with these couples reveal the unexpected fact that, although the 10 per cent. iridium alloy is rather more powerful than the rhodium alloy,

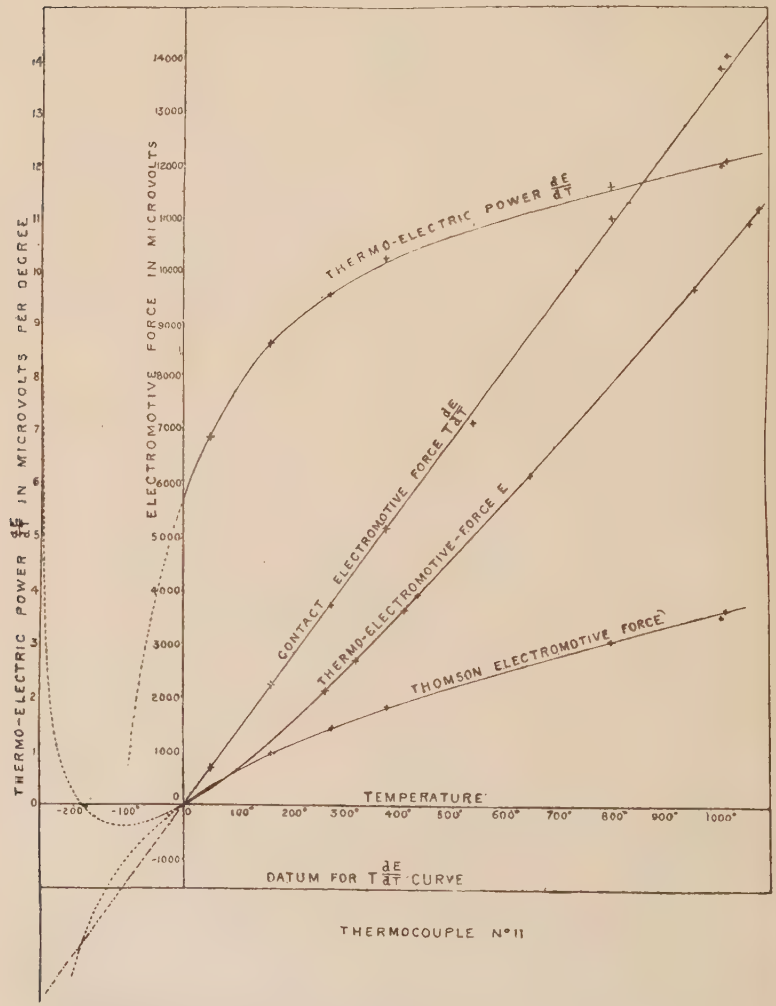
the partial substitution of iridium for rhodium in the latter alloy very materially lowers its thermo-electric power when compared with platinum. This result is of interest, as it suggests that the change in the thermo-electric power of a metal depends on the extent to which it is saturated with the alloying metal ; 10 per cent. of either rhodium or iridium would more completely saturate the platinum than would 10 per cent. of a mixture of the two metals, in the same way that a single salt more completely saturates a liquid than the same weight of a mixture of two salts of equal solubility. The curve marked " Barus, 10 per cent. iridium " was obtained by adding the mean results obtained by him with couples 19 and 20, consisting of " hard " platinum and platinum with 5 per cent. of iridium, to those obtained with couple 27, consisting of platinum with 5 per cent. of iridium and platinum with 10 per cent. of iridium. The use of " hard," *i. e.* impure platinum in these alloys probably accounts in part for the smallness of the observed thermo-electric power. A comparison of his couples 22 and 35 shows that the E.M.F. at 930° would have been about 1200 microvolts higher if " soft " platinum had been used : this, however, does not materially reduce the difference between his results and those obtained by the author with couple 13, and it appears probable that the iridium he employed contained a considerable percentage of rhodium or other metal of the platinum group.

Although the curves in fig. 5 differ so widely from one another in direction, there is a general agreement as regards their form. All the curves are convex towards the scale of temperature and their curvature decreases as the temperature rises. The 10 per cent. iridium curves are decidedly straighter in their lower parts than the 10 per cent. rhodium curves, so that their indications are more nearly proportional to the temperature.

The usual method of presenting thermo-couple observations has been adopted in plotting the curves in fig. 5, but it is evident that a more critical method is necessary; for although the uncertainties in the actual temperatures of points would be quite visible on these curves, the observations themselves can be made with an accuracy of about 1 in 10,000. Very good results are obtained by plotting the first differential of

the original curve ; that is a curve the ordinates of which represent the values of $\frac{dE}{dt}$ corresponding to the values of t indicated by the abscissæ.

Fig. 6.



This method can only be applied when a number of melting- or boiling-points have been obtained, so as to enable the value $\frac{dE}{dt}$ to be found for a sufficiently large number of points.

For rough purposes, these values are obtained by dividing the difference between the observed E.M.F.'s at two adjacent melting-points by the difference in the assumed temperatures, and plotting the result at the mid-point. A more accurate curve may be obtained by a graphic method which it is unnecessary to describe here.

In fig. 6, the thermo-electric power or $\frac{dE}{dT}$ curve has been obtained in this manner from the curve representing the thermo-electromotive force E of couple No. 11.

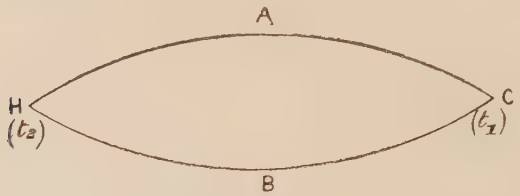
Small errors, either in the assumed temperatures of the melting-points, or in the observed values of the E.M.F. for these temperatures, are thrown into prominence by this method of plotting ; errors of a few tenths of a degree in the relative observations of the melting-points of tin, bismuth, or lead, or of one or two degrees in the melting-point of aluminium, make very distinct breaks in the curve, and although the observed values of E are all seen to lie closely on the E.M.F. curve, the calculated values of $\frac{dE}{dT}$ show in some cases a wide divergence from the smooth mean curve. Thus, for example, the observed melting-point for aluminium must have been several degrees lower than the assumed temperature, unless the curve can be admitted to have a great dip at about 550°. Records of the solidification of the aluminium employed did not show the steady freezing-temperature which is characteristic of a pure metal, and the author is inclined to attribute the low reading for aluminium (see Table II., p. 127) to the impurity of the metal rather than to irregularity of the indications of the thermo-couple.

Theory of the Thermo-Couple.

The practical value of any pyrometer depends largely on the possibility of expressing, accurately and simply, the relation between the reading of the pyrometer and the temperature to which it is exposed. In the case of the thermo-couple this has been attempted by many experimenters, partly empirically, and partly from theoretical considerations.

The simple thermo-couple HACB, consisting of two wires A and B of different metals or alloys, has its junctions H

and C at temperatures t_2 and t_1 respectively; four separate E.M.F.'s are produced: (1) a true contact E.M.F. at H, and (2) another at C, due to the difference in the nature of the two wires (the Peltier E.M.F.), and (3 and 4) an E.M.F. distributed along each of the wires A and B, due to the difference between the temperatures of different parts of the same wire (the Thomson E.M.F.).



The observed E.M.F. is, in reality, the algebraic sum of the difference between the E.M.F.'s at H and C, and the difference between the E.M.F.'s in A and B.

Lord Kelvin has shown that it is possible, by means of the second law of thermo-dynamics, to distinguish between some of these constituents of the observed E.M.F., for the thermo-couple may be regarded as a reversible engine.

Let σ be the "specific heat of electricity" in the metal A, that is, the amount of heat (measured in ergs) absorbed by the passage of one unit of electricity (or by unit current in unit time) through a rise in temperature of one degree; and let σ' be the specific heat of electricity in B.

Let H be the coefficient of the Peltier effect, that is, the amount of heat absorbed at the junction by unit current in unit time passing from A to B. Then if T_1 be the temperature of the cold junction, and T_2 that of the hot junction, measured from the absolute zero, the heat absorbed in the wires A and B in unit time by the passage of a current I will be $I(\sigma-\sigma')(T_2-T_1)$ if the temperatures T_2 and T_1 are nearly equal, so that the values of σ and σ' may be taken for a mean-temperature T.

It then follows, from the second law of thermo-dynamics, that

$$\frac{IH_2}{T_2}-\frac{IH_1}{T_1}+\frac{I(\sigma-\sigma')(T_2-T_1)}{T}=0,$$

and as

$$E = H_2 - H_1 + \int_{T_1}^{T_2} (\sigma - \sigma') dT,$$

it follows that

$$H = T\frac{dE}{dT},$$

and

$$E = T\frac{dE}{dT} + \phi(T) - C. \quad . \quad . \quad . \quad . \quad (A)$$

In this equation $T\frac{dE}{dT}$ is the E.M.F. at the hot junction, C is the E.M.F. at the cold junction, and $\phi(T)$ is $\int_{T_1}^{T_2} (\sigma - \sigma') dT$, the difference between the Thomson effects in the two wires.

No theoretical investigations have as yet determined the form of $\phi(T)$, so that it is impossible to obtain, from purely theoretical considerations, an equation connecting the temperature and the electromotive force of the couple.

Equation (A) does, however, enable us to calculate the values of the Peltier and the Thomson E.M.F.'s from a series of data connecting the observed E.M.F. and the temperature of the thermo-junction, and the results of such a calculation for the thermo-couple No. 11 are given in fig. 6.

The contact or Peltier E.M.F. of the thermo-junction is represented in the equation by $T\frac{dE}{dT}$, and may be deduced from the thermo-electric power or $\frac{dE}{dT}$ curve by multiplying each value of $\frac{dE}{dT}$ by the corresponding absolute temperature T.

The $T\frac{dE}{dT}$ curve, embodying the results of these calculations, has been plotted in the figure, the ordinates are measured from a datum-line placed at a distance below the temperature-scale equal to the E.M.F. across the cold junction; so that its ordinates from the temperature-scale give the difference between the E.M.F.'s at the hot and cold junctions. The difference between this curve and the E.M.F. curve E represents the resultant of the Thomson effects in the two wires. The curve marked "Thomson E.M.F." obtained in this way has been plotted for convenience *above* the temperature-scale

line, but it must be remembered that this E.M.F. is opposed in direction to the resultant of the contact E.M.F.'s.

It will be noticed that the curve representing the contact E.M.F., or $T\frac{dE}{dT}$, is practically a straight line. In fact, the divergences at 1000° only correspond to an uncertainty of about 2° in the assumed temperatures.

If, then, we assume that

$$T\frac{dE}{dT} = aT + b,$$

we obtain by integration $E = aT + b \log T + c$.

This equation requires three relations between E and T to determine the values of the three constants a, b, and c, and these can be accurately obtained for the temperatures 0°, 100°, and the boiling-point of sulphur (444°·53).

The temperatures given in the last column of Table II. (p. 127), were obtained in this way from the Oct 1896 results of couple 11. These temperatures really depend on an extrapolation beyond 445° of the empirical formula given above, a formula which appears to be correct in the case of this thermo-couple as far as it has been possible to verify it. The dotted lines in fig. 6 are the continuations of the curves at temperatures below 0°. They depend on the assumption that the formula holds good at these temperatures, and show that at the absolute zero the E.M.F. would become infinitely great, and that the contact E.M.F. would have a negative value of about 200 micro-volts: it has been stated * that the latter should be zero, as otherwise further cooling could be effected by the passage of a current. The $T\frac{dE}{dT}$ curve for a platinum-lead thermo-couple, calculated from figures given in Dewar and Fleming's † paper on thermo-electricity at low temperatures, is straight between 100° and 0°, but below this temperature it shows a tendency to curve in the direction required to reduce its value to nothing at the absolute zero.

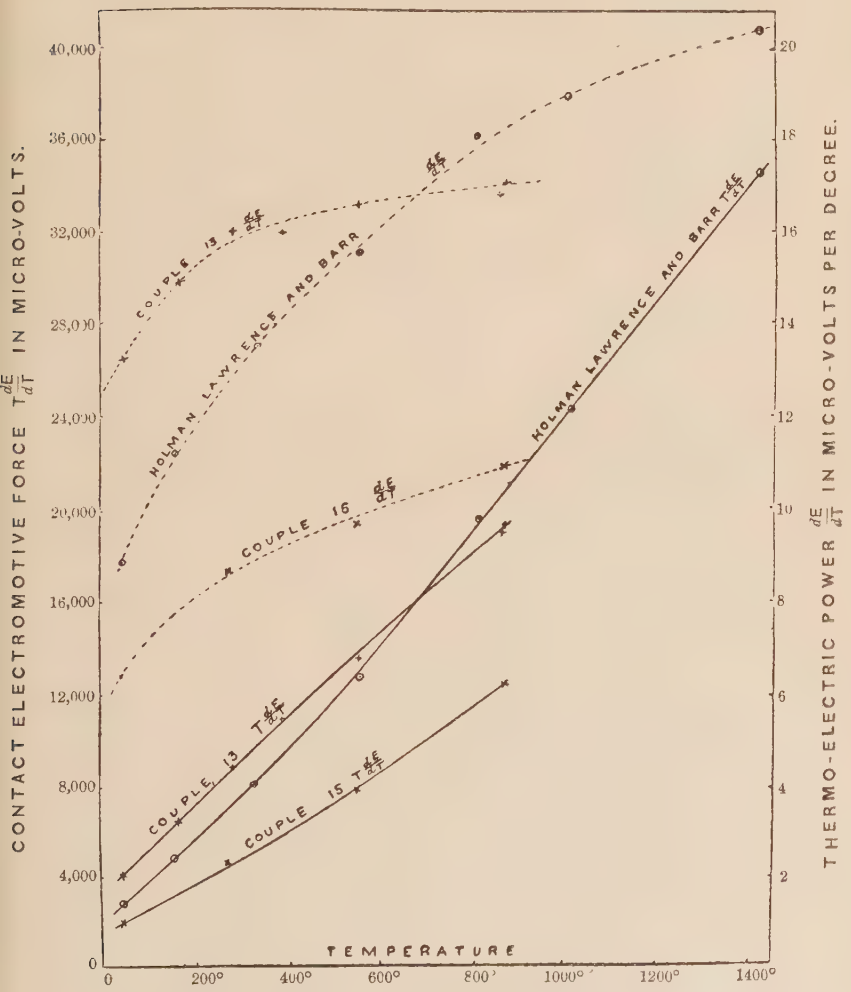
The theoretical simplicity of the formula given above, which

* Burton, Phil. Mag. vol. xxxviii. 1894, p. 66.

† Phil. Mag. vol. xl., July 1895, p. 95.

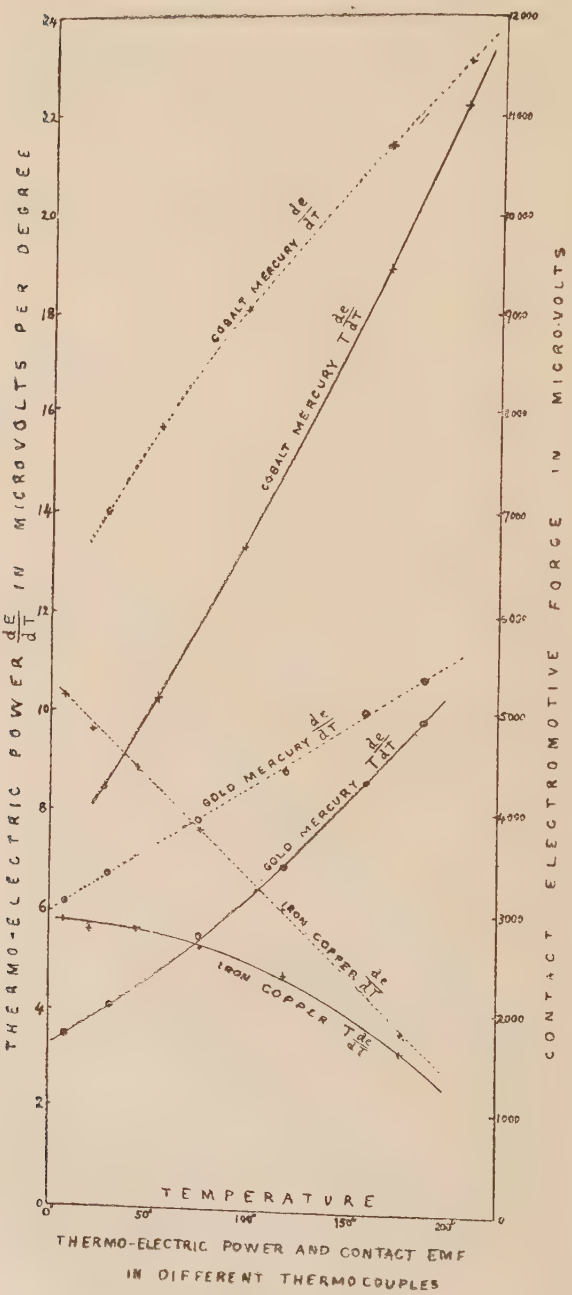
involves a linear relation between the temperature of a junction and the Peltier E.M.F., makes it worth while to consider how far this relation is true in the case of other thermo-couples. In fig. 7, which has been plotted with this object, the

Fig. 7.



THERMO-ELECTRIC POWER AND CONTACT E.M.F. IN PLATINUM THERMO-COUPLES.

Fig. 8.



THERMO-ELECTRIC POWER AND CONTACT EMF IN DIFFERENT THERMOCOUPLES

dotted lines represent the values of $\frac{dE}{dT}$, and the continuous lines those of $T\frac{dE}{dT}$, for the couples 13, 15, and 16, and for the couple used by Holman, Lawrence, and Barr.

None of the curves representing the contact E.M.F. are so nearly straight as in the case of couple 11 ; but all these curves are approximately straight, whilst those representing the values of $\frac{dE}{dT}$ are all strongly curved.

This is of great interest in connexion with the parabolic formula investigated by Tait. Tait's expression for the E.M.F. of a thermo-couple follows from equation (A) if the assumption be made that the specific heat of electricity is proportional to the absolute temperature ; but the parabolic formula is really empirical, and depends on the observations that the curves representing the values of $\frac{dE}{dT}$ are approximately straight in the case of the thermo-couples for which the formula was constructed. The formula of Tait and the formula developed in this paper are both empirical, and are both based on theoretical assumptions. In figures 6, 7, and 8, the extent to which these formulæ hold can be seen at a glance. The parabolic formula depends on the straightness of the $\frac{dE}{dT}$ curve, which is dotted in figs. 7 and 8, whilst the logarithmic formula depends on the straightness of the $T\frac{dE}{dT}$ curve.

The curves which are plotted in fig. 8 are calculated from results given in a paper by K. Noll * on "The Thermo-electricity of Chemically Pure Metals." They show that thermo-couples composed of copper, iron, gold, and mercury follow Tait's law very closely over the somewhat limited range of temperatures for which data are given, and other couples not plotted also show approximately straight lines for the values of $\frac{dE}{dT}$. A couple composed of cobalt and mercury, and another composed of platinum and copper the results of which are not plotted, show both lines curved in opposite directions, a fact

* Wied. *Ann.* 1894, p. 874.

Table I.

Electromotive Force in Micro-volts of Thermo-couples. Cold junctions at 0° C.

Substances.	Assumed temperatures.	Couple 11.				Couple 13.		Couple 15.	Couple 16.
		March 1896.	Oct. 1896.	March 1897.	Nov. 1897.	1st set, 1898.	2nd set, 1898.	Feb. 1898.	Feb. 1898.
Water boils	100	690	686	.....	.....	1324	1323	642	858
Tin melts	231·9	1811	1820	.....	1784	3320	.....	.....	.....
Bismuth melts ...	268	2171	2160	.....	2117	3929	.....	.....	.....
Lead melts	326	2723	2718	.....	.....	4840	.....	.....	.....
Zinc melts	419	3654	3652	.....	.....	6331	.....	.....	.....
Sulphur boils ...	444·53	.....	3927	.....	3894	6733	6713	3621	5245
Aluminium melts	654·5	.....	6138	.....	.....	10211	.....	5042	8528
Silver melts	960·7	.....	9686	9669	.....	.....	.....	.....	.....
Gold melts.....	1061·7	10941	10894	10938	.....	.....	17056	.....	.....
Copper melts.....	1080·5	11192	11133	11132	11106	.....	17427	10318	16536

Note.—The changes in the E.M.F. of couple 11 during two years correspond to a change of about 5° at 1100°. They were principally caused by prolonged heating in fire-clay insulators in a reducing atmosphere before the Oct. 1896 and Nov. 1897 observations. Under normal conditions the changes are very much smaller.

TABLE II.

Comparison of the Temperatures of the Melting-points of Metals afforded by recent research.

Authority. / Date of publication.	Heycock & Neville. 1895.	Holborn & Wien. 1895.	Holman, Lawrence, & Barr. 1896.	D. Berthelot. 1898.	The Author. 1898.
Instrument	Resistance Pyrometer.	Thermo-couple.	Thermo-couple.	Interference Method	Thermo-couple no. 11.
Calibration data.	0° 100° 444°·53	Porcelain air-thermometer.	0° 444°·53 100° 1072° 218°·2	Expansion of air.	0° 100° 444°·53
Tin	231°9	... ..	.....	.....	232°1
Bismuth	.....	.....	.....	.....	268·4
Lead	.....	.....	.....	.....	325·9
Zinc	418·96	.....	.....	.....	418·2
Aluminium	654·5	.....	660°	.....	649·2
Silver	950·7	968°	970	962°	961·5
Gold	1061·7	1072	(1072)	1064	1062·7
Copper	1080·5	1082	1095	.....	1083·0

which suggests that thermo-electrically there may be two classes of metals:—(1) the ordinary metals, for which the $\frac{dE}{dT}$ curves are straight; and (2) the platinum metals, together with a few other metals such as nickel and cobalt, for which the $T\frac{dE}{dT}$ curves are straight. A couple composed of one metal from each class would of course give both lines curved. On the other hand, it is unlikely that there would be any sharp division of this kind, and both formulæ may be particular cases of a more general one.

In view of the curvature exhibited by the $\frac{dE}{dT}$ curves of thermo-couples composed of platinum and platinum alloyed with iridium or rhodium, there can be no object in applying

the parabolic formula to their indications, though this has been done by several experimenters.

In conclusion, the author wishes to express his thanks to Prof. Roberts-Austen for the helpful interest he has taken in the research.

Discussion.

Dr. Chree discussed the curves and asked how far stirring affected the results; he was inclined to think that stirring was a mistake.

Mr. A. Campbell enquired whether the galvanometer kept its zero sufficiently well throughout the tests.

Mr. Stansfield, in reply, said he had also come to the conclusion that stirring was a mistake; it was a mistake to use a large quantity of metal. The pyrometers were sensitive to about a tenth of a centigrade degree. He had experienced great difficulty with regard to the zero of the galvanometer.

PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY

OF LONDON.

SESSION 1897-98.

February 26th, 1897.

Shelford Bidwell, Esq., President, in the Chair.

The following was elected a Fellow of the Society :—

Prof. K. Pearson, F.R.S.

The following communications were made :—

"On the Photography of Ripples." By Mr. J. H. Vincent.

"On the Thermoelectric Properties of some Liquid Metals." By Mr. W. B. Burnie.

March 12th, 1897.

Shelford Bidwell, Esq., President, in the Chair.

The following was elected a Fellow of the Society :—

Dr. J. H. Harker.

The following communication was read :—

"On a Mechanical Cause of Homogeneity of Structure and Symmetry Geometrically Investigated; with Special Application to Crystals and to Chemical Combination." By Mr. William Barlow.

March 26th, 1897.

Meeting held in the Physical Laboratory of the Finsbury Technical College by invitation of Prof. S. P. Thompson.

Shelford Bidwell, Esq., President, in the Chair.

The following were elected Fellows of the Society:—
Dr. Dawson Turner and Mr. James Quick.

Mr. R. Appleyard exhibited some experiments on Liquid Coherers and Mobile Conductors.

Prof. Dalby exhibited the following pieces of apparatus:—

1. Ewing's Reading Telescope.
2. A Kinematic Slide.
3. An Inertia Apparatus with trifilar suspension.
4. A Kinematic Hook-Gauge.
5. A Wilberforce Spring.

Prof. S. P. Thompson exhibited the following pieces of apparatus:—

1. A Model illustrating the Synthesis of two Opposite Circular Motions of equal Period and Amplitude.
2. A Model illustrating the Synthesis of two Simple Harmonic Motions of equal Period and Amplitude with any difference of Phase to form Circular Motion.
3. The Projection of Arago's Rotations.
4. Some experiments with Heat-indicating Paint.
5. A new Model illustrating the Propagation of a Hertz Wave.

April 9th, 1897.

Shelford Bidwell, Esq., President, in the Chair.

Mr. Garrett and Mr. Lucas communicated a paper "On a Nickel Stress Telephone."

Mr. W. A. Price communicated a paper "On Alternating Currents in Concentric Conductors."

Mr. W. B. Morton communicated a paper "On the Effect of Capacity on Stationary Electric Waves in Wires."

May 14th, 1897.

SHELFORD BIDWELL, Esq., President, in the Chair.

The following was elected a Fellow of the Society :—

Mr. F. R. FLEW.

Mr. WATSON exhibited an Instrument for the Comparison of Thermometers.

Dr. MORRIS communicated a paper "On the Effect of Temperature on the Magnetic and Electrical Properties of Iron."

Mr. APPLEYARD read a paper "On the Formation of Mercury Films by an Electrical Process."

May 28th, 1897.

SHELFORD BIDWELL, Esq., President, in the Chair.

The following was elected a Fellow of the Society :—

Mr. G. THOMAS-DAVIES.

Dr. A. A. GRAY communicated a paper "On the Perception of the Difference of Phase by the two Ears."

Mr. J. ROSE-INNES read a paper "On the Isothermals of Iso-pentane."

June 11th, 1897.

SHELFORD BIDWELL, Esq., President, in the Chair.

The following were elected Fellows of the Society :—

Messrs. R. H. HOUSMAN and F. W. LANCHESTER.

Mr. C. S. WHITEHEAD read a paper "On the Effect of Sea-Water on Induction Telegraphy."

Mr. BLAKESLEY read a paper "On a New Definition of Focal Length, and an Instrument for its Determination."

Drs. J. E. MYERS and F. BRAUN communicated a paper "On the Decomposition of Silver Salts under Pressure."

Prof. J. A. FLEMING read a paper "On a new way of Determining Hysteresis in Straight Iron Strips."

June 25th, 1897.

Shelford Bidwell, Esq., President, in the Chair.

Mr. Sutherland communicated a paper "On a New Theory of the Earth's Magnetism."

Dr. Kuenen read a paper "On Experiments in Critical Phenomena."

Dr. Barton communicated a paper "On the Attenuation of Electric Waves in Wires."

Mr. Searle read a paper "On the Steady Motion of an Electrified Ellipsoid."

October 29th, 1897.

Shelford Bidwell, Esq., President, in the Chair.

Prof. Stroud described the Barr and Stroud Naval Range-Finder, and a Telemetrical Focometer and Spherometer.

Mr. A. S. E. Ackermann exhibited some Experiments illustrating Surface-Tension.

November 12th, 1897.

Dr. G. Johnstone Stoney, Vice-President, in the Chair.

Mr. J. Rose-Innes read a paper "On the Isothermals of Ether."

Messrs. F. S. Spiers, F. Twyman, and W. L. Waters communicated a paper, "On the Variations of the Electromotive Force of the H-form of Clark Cells with Temperature."

November 26th, 1897.

Shelford Bidwell, Esq., President, in the Chair.

The following was elected a Fellow of the Society:—

Mr. S. W. J. Smith.

Mr. R. Appleyard read a paper "On the Failure of German Silver and Platinoid Wires."

Prof. Perry read a note on Thermodynamics.

December 10th, 1897.

SHELFORD BIDWELL, Esq., President, in the Chair.

Mr. A. CAMPBELL showed two simple Lecture Experiments and an Apparatus for the Self-acting Temperature Compensation of Standard Cells.

Mr. J. ROSE-INNES read a paper "On Lord Kelvin's Absolute Method of Graduating a Thermometer."

January 21st, 1898.

SHELFORD BIDWELL, Esq., President, in the Chair.

The following were elected Fellows of the Society :—

Messrs. C. O. BARTRUM, and W. FRANCIS, Jun.

Prof. FITZGERALD exhibited some Photographs showing the Zeeman Effect which had been taken by Mr. Preston.

Prof. LODGE opened a discussion on Electrical Signalling without Connecting Wires.

Prof. S. P. THOMPSON exhibited a new form of Tesla Oscillator.

Annual General Meeting.

February 11th, 1898.

SHELFORD BIDWELL, Esq., President, in the Chair.

The following were elected Fellows of the Society :—

Messrs. G. S. ECKFORD and Prof. A. LODGE.

The following Report of the Council was read by the Secretary: —

During the past Session the work of the Physical Society has been carried on steadily and satisfactorily.

Thirteen meetings have been held at which papers have been read. One of these, on March 26th, 1897, was held at the Finsbury Technical College, Leonard Street, City Road, by the kind invitation of the Foreign Secretary, Prof. S. P. Thompson. There was a large attendance of Fellows and Visitors, and great interest was felt in

Prof. Thompson's laboratory. All the other meetings were held in the rooms of the Chemical Society at Burlington House.

Eleven new Fellows have been elected into the Society, and the Council regrets to report that among those who have been lost to the Society through death are Lord Sackville Cecil, the Rev. H. Freeman, Mr. Adam Hilger, and Mr. J. W. Rodger. Short memoirs of these will be published in the 'Proceedings.' There have been two resignations, so that the numbers of the Society are more than kept up.

The only other matter that calls for special notice is the completion of the negotiations with the Institution of Electrical Engineers. The result of this will be that the combined strength of the two Societies will be devoted to the perfection of the system of Abstracts of Physical Papers already carried on by this Society for the past three years.

Lord Sackville Arthur Cecil was the son of the second Marquess of Salisbury by his second wife, now the Dowager Countess of Derby, and was born in 1848.

His education was directed by his father, who in 1858–9 was Lord President of the Council, and thus had charge of what is now the Education Department. The late Lord Salisbury much studied the subject of education, and had views of his own as to the manner in which boys should be brought up. As a consequence, Lord Sackville was accustomed to use his hands from his childhood, and the habit of observation thus acquired was most valuable to him in afterlife. He was very methodical and acted up to the maxim: "A place for everything, and everything in its place." He was a pupil at Wellington College for a short time in the early days of the school, and there his study was a type of neatness; he had a cabinet of tools, and if any of them were not in their proper places he seemed quite distressed. This methodical habit was continued through life, and those who knew the study in his house at Hayes Common will remember the perfect order of its arrangement.

After leaving Wellington College, Lord Sackville was placed under the care of a private tutor, and about this time he made a practical acquaintance with the working of the electric telegraph

in the signal-box at Hatfield Station, and he obtained a pair of single-needle instruments, by which he connected two distant rooms at Hatfield House. Later, he and his two younger brothers went to Paris with his tutor for a year, and there, for the first time, he saw an electric bell, which so much pleased him that he introduced them into the house on his return home. At the early age of 14 he devised an ingenious mode of connecting by electricity the turret clock at Hatfield with a clock in the house, which was driven by the current. He afterwards spent more than four years with the same private tutor at the old Manor House of Cranborne; here he was prepared for entering the University of Cambridge; he matriculated at Trinity College as a Fellow Commoner in 1865, and took his degree in Applied Science in 1869.

While still an undergraduate Lord Sackville Cecil was appointed an Associate Juror for this country at the Paris Exhibition in 1867 for Telegraphic Apparatus and Processes, Sir Charles Wheatstone being the Juror for that Class.

After leaving Cambridge, Lord Sackville worked in the Great Eastern Railway workshops at Stratford, and also in those of the Great Northern Railway at Doncaster. For a short time he was attached to the engineers' staff of the French Atlantic Cable, and subsequently acted as chief electrician in the laying of the Marseilles-Bona Cable.

From 1878 to 1880 he was Assistant General Manager of the Great Eastern Railway, and General Manager of the Metropolitan District Railway from 1880 to 1885.

After retiring from the latter post, which he found very arduous, Lord Sackville Cecil again devoted his attention to electrical matters, and became a Director of the Eastern Telegraph and of the Brazilian Telegraph Companies. He died on January 29, 1898.

He was an enthusiast for work and was most conscientious in the performance of all his duties, and no consideration of personal comfort, or even the opinion of others, would deter him from doing what he believed to be right. He made himself acquainted with all the details of the work he had to direct, so that in the event of anything going wrong he should be able at once to properly correct it. Although it does not appear that he published any original investigations, yet this was probably more due to an innate diffidence than to any other cause. He always showed an intense interest in all scientific subjects, and many would be surprised to

hear of the numerous applications of physical and chemical science which he made at different times as occasions served.

He was one of those men whose true value we begin to learn after they are taken from us.

Lord Sackville Cecil was one of the original members of the Physical Society.

The Rev. Alexander Freeman was born January 28th, 1838, and was educated at the Merchant Taylors' School, whence he proceeded to St. John's College, Cambridge, in 1857, having obtained a Scholarship on the Foundation. He graduated a Fifth Wrangler in 1861, and in February 1862 he gained the Chancellor's Gold Medal for Legal studies. He was elected a Fellow of his College in May 1862, and took his M.A. degree in 1864. He acted as deputy for the Plumian Professor of Astronomy at Cambridge from 1880 to 1882. He translated and compiled an English edition of Fourier's *Analytical Theory of Heat*, and added to it many valuable original notes, and recently brought out a new edition of Cheyne's *Planetary Theory*. To the *Monthly Notices* of the Royal Astronomical Society, of which he was elected a Fellow in 1864, he contributed about a dozen papers. He further acted as Director of the Saturn Section of the British Astronomical Association, from which office he retired, on account of failing health, in 1895. His death took place somewhat suddenly on Saturday, June 12, 1897. He was admitted a Member of the Physical Society during the first session. In addition he was a Fellow of the Cambridge Philosophical Society, and a Member of the London Mathematical Society.*

By the death at the early age of 29 of James Wyllie Rodger the Society has lost one of its most promising Fellows. Although he was a comparatively newly-elected Fellow, having joined the Society in 1895, he always took part in the discussions on any papers of a physico-chemical nature read before the Society, and so was known to most of the Fellows.

He was born at Stewarton, N.B., on December 11, 1867, and was educated at Kilmarnock Academy, and at the Royal College of Science, South Kensington. His career as a student was interrupted by illness, but nevertheless he won all the chief prizes and

* This note is an abstract of one appearing in the *Proceedings* of the London Mathematical Society.

besides took a very prominent place in the management of the College societies. In 1889 he was appointed an Assistant in the research laboratory of the Chemical division of the Royal College of Science, and during the next few years he was the joint author with Dr. Thorpe of a number of papers bearing on Chemical Physics. The most important of these were on the Viscosity of Liquids, and were published in the Transactions of the Royal Society. One of these papers was selected for the Bakerian lecture in 1894. The lecture was delivered by Rodger, and all who were present will have a lively recollection of the very admirable manner in which it was delivered. Although he was the youngest Bakerian Lecturer, there can be no doubt that the lecture had rarely, if ever, been better delivered. During the evenings of the years 1893-5, while his teaching work occupied his whole time during the day, Rodger was spending his evenings working with his friend Mr. W. Watson on the magnetic rotation of the plane of polarization of light in liquids, the results of the first part of the work being published in the Transactions of the Royal Society. It is to be feared that working as he did at this time, often till late into the night, was in a certain measure responsible for his subsequent breakdown.

In addition to his experimental work Rodger was a frequent contributor to *Nature* and *Science Progress*, his series of papers on the "New Theory of Solutions" in the latter being particularly noticeable.

Of singularly attractive appearance and manners, popular with his fellows, a good teacher, and a first-rate lecturer, he had done enough solid work to prove that, if life and health had been spared, he would have won an honourable place in the ranks of English science. This was, however, not to be, for early in 1896 he was attacked by influenza, and from this he never recovered. He lingered on for about a year and then died, on May 5, 1897.

The Report of the Council was adopted.

The following were elected Honorary Fellows of the Society:—

Prof. Ricardo Felici and Prof. Emilio Villari.

The election of Officers and other Members of Council then took place, the new Council being constituted as follows :—

President.—Shelford Bidwell, M.A., LL.B., F.R.S.

Vice-Presidents who have filled the Office of President.—Dr. J. H. Gladstone, F.R.S.; Prof. G. C. Foster, F.R.S.; Prof. W. G. Adams, M.A., F.R.S.; Lord Kelvin, D.C.L., LL.D., F.R.S.; Prof. R. B. Clifton, M.A., F.R.S.; Prof. A. W. Reinold, M.A., F.R.S.; Prof. W. E. Ayrton, F.R.S.; Prof. G. F. Fitzgerald, M.A., F.R.S.; Prof. A. W. Rücker, M.A., D.Sc., Sec.R.S.; Capt. W. de W. Abney, R.E., C.B., D.C.L., F.R.S.

Vice-Presidents.—C. Vernon Boys, F.R.S.; Major-General E. R. Festing, R.E., F.R.S.; G. Griffith, M.A.; Prof. J. Perry, D.Sc., F.R.S.

Secretary.—H. M. Elder, M.A.

Foreign Secretary.—Prof. S. P. Thompson, D.Sc., F.R.S.

Treasurer.—Dr. E. Atkinson.

Librarian.—W. Watson, B.Sc.

Other Members of Council.—Prof. H. E. Armstrong, D.Sc., F.R.S.; Walter Baily, M.A.; L. Clark, C.E., F.R.S.; A. H. Fison, D.Sc.; R. T. Glazebrook, M.A., F.R.S.; Prof. A. Gray, LL.D., F.R.S.; Prof. J. Viriamu Jones, M.A., F.R.S.; S. Lupton, M.A.; Prof. G. M. Minchin, M.A., F.R.S.; J. Walker, M.A.

Votes of thanks were passed to the Auditors, the Officers and Council, and to the Chemical Society.

The President then gave the following Address :—

Address by the President,

SHELFORD BIDWELL, M.A. LL.B. F.R.S.

It has been thought desirable by the Council that the proceedings at the Annual General Meeting of the Physical Society should include an Address by the President.

The form assumed by the presidential address in the case of other scientific bodies varies greatly. In the absence of any definite precedent, I propose on this occasion to begin with a few words upon the history and objects of our Society, and afterwards to glance briefly at the principal events of physical interest which have occurred during the past year.

The Physical Society was founded in 1874, and owed its origin mainly to the initiative of the late Prof. Guthrie. From the first years of its existence up to the present time it has included among its members nearly all the leading physicists of the United Kingdom.

The principal objects of the Society, as stated in the Memorandum of Association, are to receive and discuss written or oral communications relating to Physics; to invite the exhibition of apparatus for physical research and of experiments illustrative of physical laws and phenomena, and to print and publish any communications made to the Society and other matter relating to Physics.

In the early days our meetings were, by permission of the Lords of the Committee of Council on Education, held in the Physics lecture-room of the Royal College of Science, at 3 o'clock on Saturday afternoons, members being allowed the free use of the laboratory apparatus for the illustration of their papers. The proceedings were at that time rather less formal than is customary at present. The papers communicated were rarely if ever "referred" before being read; often, indeed, they were read long before they were actually written, while a large proportion of the communications were of a purely oral character and never intended for publication at all, except perhaps in the short notes which the reporter sent to the scientific journals. Special prominence was given to experimental demonstrations illustrative not only of original researches but also of such work, carried on outside the Society, as happened to be attracting attention at the time; and many little devices were exhibited which could be employed for purposes of lecture illustration or class instruction.

Under the somewhat lax *régime* which then prevailed it necessarily happened that the communications made to the Society were not always of a very high order of merit, and, in fact, they were sometimes blemished by serious errors. But the demolition of the authors added to the interest and liveliness of the discussions, and no one was much the worse. For from the very beginning the Council has always been careful not to print in the 'Proceedings' anything that was not of sound scientific value, and while the number of important papers that have been published through the medium of the Physical Society is large, very little of doubtful quality has had a place in the Journal.

The first material change in our routine took place in the year 1889, when the day and hour of meeting were altered from Saturday afternoon at 3 to Friday at 5 o'clock. This was done primarily with the object of meeting the convenience of the younger members of the Society, who, it was thought, would show a greater devotion to science if they were not called upon to sacrifice their Saturday half-holidays to its claims.

I may here mention that the Council has more than once seriously considered whether it might not be expedient to hold our meetings in the evening. Many no doubt would consider this preferable, but the balance of convenience appears to be clearly in favour of the afternoon; and it is worthy of remark that at the Royal Society, as recently testified by the President, the attendance has largely increased since the meetings have been held at half-past four instead of eight o'clock.

While domiciled at South Kensington the Physical Society published and presented to its members a number of valuable books, including, among others, Prof. Everett's well-known treatise on the C.G.S. System of Units and the works of Wheatstone and of Joule. It also issued twelve volumes of 'Proceedings,' in which were collected such of the communications to the Society as had been approved for publication. By an arrangement with the proprietors of the 'Philosophical Magazine,' the same papers were also (as now) printed in that journal, being thus, to the authors' great advantage, assured of a wide circulation throughout the scientific world.

In the same period, notwithstanding the small amount of the annual subscription paid by members and of the composition fee for life membership, the Society's income so far exceeded its expenditure that it was able to accumulate and invest a capital of nearly £3000.

When the Society entered upon its twenty-first year with a position which, if somewhat unpretending, was well recognized and firmly established, it was felt that the time had come when in the interests of physical science something more than had been already achieved might fairly be demanded of it. British physicists had long been at a serious disadvantage, in that they were without any means of readily ascertaining what was being done by their fellow-workers in other countries; with the multiplication of scientific literature the need of some periodical digest similar in character to the German *Beiblätter* was becoming year by year more urgent. To endeavour to meet this want was a duty which clearly devolved upon the Physical Society, and the Council anxiously considered the question whether the publication of monthly abstracts of physical papers appearing in foreign journals could be undertaken by ourselves.

The only serious objection to the enterprise was of a financial nature. The work, if it were to be carried out efficiently, would certainly necessitate an annual expenditure exceeding by some hundreds of pounds the total income of the Society. This could only be met by raising the amount of the annual subscription and composition fee, which, as I have mentioned, were unusually low. But it is a delicate and difficult matter to ask existing members of a society for increased subscriptions unless very excellent reasons can be shown for the demand. The Council therefore determined that they would publish a series of Abstracts for one year at least before taking any steps to provide additional income, defraying the cost from cash in hand, and, if need should arise, drawing upon the invested capital. In this way it was hoped to convince members of the utility of the undertaking which they were to be called upon to support.

I need not remind you of the highly satisfactory result of the experiment. The work of the able and assiduous body of abstractors, whose names appear on the cover of our 'Proceedings,' was on all sides cordially approved; and at a Special General Meeting, held in 1896, a resolution submitted to the Society for increasing the subscription to £2 2*s*. per annum was passed almost, if not quite, unanimously. The number of those who in consequence of this increase have resigned their membership has been unexpectedly small, while on the other hand many of the life-members have, in response to the invitation issued to them, voluntarily contributed an additional £15 15*s*. to the funds of the Society, in recognition

of the fact that they are now in enjoyment of greater and more costly advantages than were contemplated at the time when they paid their very moderate composition fees. To such as have not yet responded, I venture to repeat the invitation.

Although the Abstracts were actually published for two years before the increased subscriptions began to come in, the whole cost was met out of uninvested cash supplemented by grants liberally made by the British Association and the Royal Society, and it was never found necessary to draw anything at all from the reserve fund. I wish to emphasize this fact because the abundant caution properly exercised by the Council in entering upon a new and uncertain enterprise appears to have led to a very general impression that the Society had outrun its means and was on the verge of bankruptcy, whereas, in truth, it was never in a more prosperous financial condition than it is at present.

On October 26th, 1894, the Society met for the first time in this room. Although the privileges so generously accorded to us by the authorities at South Kensington were highly valued, it was nevertheless deemed advisable that we should leave the home of our youth and seek a footing in Burlington House, the headquarters of scientific associations. Here the Chemical Society offered us a most kindly and cordial welcome and provided us with a meeting place which is not only more generally accessible than the old one, but is also on other grounds more convenient to most of our Fellows.

By this time the general affairs of the Society had assumed a more business-like condition. Amongst other things greater care was exercised with regard to the acceptance of communications. I need hardly say that no paper is in these days allowed to be read unless it has been first referred to some competent authority and favourably reported upon. The most distinguished physicists in the kingdom have given their services as referees, and our heartiest thanks are due to them for the care and patience which they have ungrudgingly bestowed upon a somewhat ungrateful task.

The practice was also introduced of putting the more important papers into type before they were read and distributing the proofs among such of the Fellows as were known to be specially interested in the subjects to which they related. This course has led to a considerable improvement in the value of the discussions.

The most recent step in advance consists in the adoption of a scheme for greatly extending the list of journals from which the monthly abstracts are made. With regard to this scheme, which

was finally approved by the Council at the meeting three weeks ago, I shall say a few words presently.

In reviewing the history of the Society for the past 24 years, there occurs to me only one material point in respect of which the old times were better than the new. It must be confessed that there has been a lamentable falling off from the abundance of experimental illustration, which in former days added so greatly to the attractiveness of our meetings. The remedy lies chiefly in the hands of our Fellows. It is true we have not the same facilities now as when the resources of South Kensington were at our disposal, and that an experimental demonstration may consequently involve more trouble and, perhaps, some additional cost. But with regard to the latter objection I have very little hesitation in saying, though I am quite without authority to do so, that, if necessary, payment out of the Society's funds of the reasonable expenses incidental to any appropriate demonstration would be readily sanctioned by the Council and approved by the Society generally. Let me remind Fellows that the exhibition of experiments or of apparatus is not necessarily accompanied by the reading of a formal paper, and that the experiments need not even be new, in the sense that they have never been exhibited elsewhere. Anything of general interest to physicists would always be heartily welcomed.

Among the objects for which the Physical and other kindred societies are established is a very important one to which no reference is made in the Memorandum of Association. One of the most useful functions of these institutions is to bring together and promote friendly intercourse among fellow-workers in a particular branch of science. In this connection I myself (if you will pardon me for referring to personal matters) owe a heavy debt of gratitude to the Physical Society. At the time when I was desirous of becoming a member, I was not personally known to a single man who was in a position to support my candidature. After some preliminary correspondence I introduced myself to Prof. Roberts-Austen, then one of the secretaries; he kindly gave me an introduction to Prof. Adams, the President, and the two were good enough to sign my recommendation form. Who furnished the third necessary signature I never ascertained. In spite of this somewhat inauspicious *début*, it was my good fortune after, and solely as the result of, a few years' more or less regular attendance at the meetings, to have made a large number of acquaintances, and, I may say, very good friends, among the leaders and workers in science. I have long regarded my connection with the Physical

Society as the source of one of the chief interests of my life; and for the highly valued honour you have done me in electing me to be your President I cannot sufficiently express my thanks.

It was mainly with the view of promoting social intercourse among the Fellows that the experiment was tried three years ago of providing tea before the meetings. I regret to say that the result was a distinct failure. For some reason or other the institution was never appreciated, and when it was found that, owing to the scanty attendance, the cost of a cup of tea had reached an exorbitant amount the luxury had to be abandoned.

I believe now that this want of success was the consequence of an oversight in the arrangements. In order to economise time Members of Council were served with tea in the room where they transacted the Society's business. This was a mistake. They ought to have gone upstairs with the rest of us and fulfilled the duties of hosts or rather, perhaps, of masters of the ceremonies. I hope that in the future the experiment may be repeated under the more favourable conditions which I have indicated.

I must refer to one other very efficient means of encouraging sociability among our members. Through the kindness of certain influential gentlemen belonging to the Society, we have from time to time been afforded the opportunity of holding a meeting in some well-known physical laboratory either in town or in the country. I myself have had the privilege of taking part in most agreeable pilgrimages to Oxford, Cambridge, and Bristol, and have also been among those who enjoyed the hospitality of Profs. Adams, Carey Foster, Ayrton, and Thompson at their laboratories in London. It gives me great pleasure to be able to say that the Council has accepted an invitation from our Fellow, Mr. Porter, to go to Eton for the next meeting, which will be held on Saturday, February 26th, and I trust that our appreciation of his kindness will be testified by a large gathering.

It is a little difficult to beg for favours, but I may be allowed to suggest to those who are in a position to exercise similar hospitality, that they have it in their power to contribute in a material degree and in more than a merely scientific sense to the wellbeing of the Society.

And now I pass to the second of my proposed topics.

Perhaps the most remarkable event of purely physical interest which has attracted attention during the past year is the discovery of the Zeeman effect. Towards the end of 1896 Dr. Zeeman of Amsterdam made the observation that the spectral lines characteristic of certain elements in a state of vapour, became broadened

if the vapours were submitted to the influence of a strong magnetic field. Later experiments showed that the lines, instead of being merely increased in breadth, could be actually divided. A single line, if seen in the direction of the magnetic force, became in many cases split into two, which were circularly polarised in opposite senses; while, if looked at across the lines of force, it was tripled, the two outer lines being plane-polarised in one direction and the middle line in the direction at right angles. The phenomenon has been shown to be consistent with and confirmatory of the hypothesis that the ions carry electrical charges. Exceedingly beautiful photographs of the Zeeman effect in zinc, cadmium, and iron, made by Mr. Thomas Preston, were exhibited by Prof. Fitzgerald at our last meeting.

Among the most important of the papers which have been published by our own Society is Dr. Chree's memoir on the Applications of Physics and Mathematics to Seismology, which ran through three numbers of our 'Proceedings.' The author has discussed from a theoretical standpoint the influences exerted by such agencies as ocean and estuary tides, changes of barometric pressure, and the gravitational action of the moon and the sun, in producing temporary distortion at the earth's surface. He has also given numerical examples and indicated the most favourable conditions for the direct observation of some of the effects.

Another paper of high scientific interest and possible technical value is that on the Magnetic Properties and Electrical Resistance of Iron as dependent upon Temperature, by Dr. David Morris. Dr. Morris's results may constitute an important contribution towards the physical data required for the solution of the mystery presented by the magnetic metals.

A paper representing a large amount of careful experiment, on the Condensation and Critical Phenomena of some Substances and Mixtures, was communicated by Dr. Kuenen.

Mention should also be made of Mr. Vincent's skilful work in connection with the instantaneous photography of mercury ripples. Several of the beautiful pictures which were exhibited upon the screen when his paper was read have been successfully reproduced in the July number of the 'Proceedings.'

A matter of the greatest interest to all physicists is the appointment of the National Physical Laboratory Committee, with Lord Rayleigh as Chairman. The terms of reference are as follows:—"To consider and report upon the desirability of establishing a National Physical Laboratory for the testing and verification of

instruments for physical investigation: for the construction and preservation of standards of measurement: and for the systematic determination of physical constant and numerical data, useful for scientific and industrial purposes. And to report whether the work of such an institution, if established, could be associated with any testing or standardising work already performed, wholly or partly at the public cost."

Prof. Silvanus Thompson and myself were appointed by the Council to give evidence before the Committee on behalf of the Physical Society.

The importance of accurate measurement in scientific research is sufficiently obvious. The value of much experimental work would be increased, much that is now practically impossible would become easy, and much valuable time would be saved if investigators could send their measuring instruments to be tested and certified by an undoubted authority.

Again, the important work of determining the physical properties or constants of matter should not be left to the caprice of volunteers. Such work is often of a tedious and uninteresting character; if it is to be of value, expensive apparatus is needed, and the operator must in many cases possess skill of a special kind; hence it is not readily undertaken unless there is some immediate object in view or some commercial purpose to be served.

The consequence is that our available tables of physical constants are of very unequal value. While in some parts they are excellent, in others they are seriously defective. The numbers given by various authorities often differ by as much as 10 per cent., and there are many gaps still remaining which ought to be filled up.

The work of enlarging and revising such tables should undoubtedly be undertaken in a systematic way by a national institution. No attempt should be made to render it directly remunerative, and little, if any, regard should be had to the possible utility of the results; it is impossible to foresee their value. Much of this work might perhaps be advantageously carried out in co-operation with similar institutions on the Continent.

The germ of such a laboratory as is desired already exists at Kew, and a better site could not easily be found. It has the advantage of being within easy reach of London and at the same time is almost entirely free from disturbances due to heavy traffic and the commercial applications of electricity.

There is reason to hope that this urgent national want will at

last receive at the hands of the Government the attention which it has so long deserved.

An event of last year which may, perhaps, be regarded by our Fellows with mixed feelings, is the foundation of a Röntgen Society under the presidency of Prof. Silvanus Thompson. The new Society has no doubt a wide field open to it in relation to matters physiological and medical and others which could not properly be included within the domain of Physics, and in such directions everyone must wish it a prosperous career. But if any investigator—even Prof. Thompson himself—should in the future make a notable discovery as regards the physical nature and properties of X-rays, I venture to express the hope that he will consider our own Society to be the proper recipient of his communication.

The publication by our Society of the highly-appreciated Abstracts was continued in 1897 upon the same lines as in the two preceding years, the papers abstracted being exclusively such as had been published in foreign journals and were of primarily scientific interest. In the present year, as the result of an arrangement with the Institution of Electrical Engineers, the number of the abstracts is to be greatly increased, British publications and papers of a technical character being included in their scope.

The arrangement in question is open to the objection that it entails the loss of our monopoly in the publication, for members of the Institution of Electrical Engineers will, like ourselves, receive copies of the Abstracts and will share with us whatever credit attaches to their production. The objection, however, appears to be in the main only a sentimental one and of small weight in relation to the substantial advantage accruing to our members—an advantage which could not possibly have been provided out of our own unaided resources.

I am happy to be able to announce that the work will still be carried on under the able editorship of Mr. Swinburne, with the assistance of a committee consisting of members of the two Societies interested. The Physical Society is under great obligations to Mr. Swinburne for the care and skill which he has for the last three years devoted to a laborious task, and to which is largely due the high reputation at present enjoyed by our Journal. There can, I think, be no reasonable doubt that under his guidance our latest venture will conduce to the continued welfare and prosperity of the Physical Society.

The Treasurer in Account with the Physical Society, from January 1st, 1897, to December 31st, 1897.

Dr.	£	*s.*	*d.*	£	*s.*	*d.*
Jan. 1, 1897. Balance in the Bank				154	6	3
Entrance-Fees	11	0	0			
Subscriptions for 1896	3	0	0			
„ 1897	9	0	0			
„ 1897	310	16	0			
„ 1898	2	2	0			
				335	18	0
Extra Subscriptions				32	10	0
Dividends				81	12	8
Grant from the British Association	100	0	0			
„ „ Royal Society	50	0	0			
				150	0	0
Sales				61	8	3
Advertisements				9	10	10
Return of Income Tax				7	1	0
Cheque passed through Treasurer's private account				16	0	0
Due to Treasurer				2	17	2
				£851	4	2

Cr.	£	*s.*	*d.*	£	*s.*	*d.*
Jan. 1, 1897. Balance paid to the Treasurer				3	4	10
Abstracts :—						
Editor	50	0	0			
„ Petty Cash	3	3	8			
Abstractors	72	11	7			
				125	15	3
Williams and Norgate (Periodicals)				18	10	10
Name Plate				2	2	0
Chemical Society				20	16	6
Attendance				2	5	0
Binding				2	6	6
Reporting				22	18	7
Bank Commission on Cheques				0	3	0
Taylor and Francis :—						
Proceedings	175	12	6			
Abstracts	215	12	6			
Miscellaneous	59	3	3			
Separate Copies	13	6	0			
				463	14	3
Petty Cash :—						
Mr. Elder	5	4	1			
Mr. Blakesley		12	1			
Dr. Atkinson	2	5	1			
				8	1	3
Deposit Account				21	1	0
Balance in the Bank, Dec. 31, 1897				160	5	2
				£851	4	2

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ASSETS.	£	s.	d.
Subscriptions due, estimated	16	0	0
£400 Furness 4 per cent. Debenture Stock	456	0	0
£1000 Midland 4 per cent. Preference Stock	1194	0	0
£200 Metropolitan Board of Works 3½ per cent. Stock	217	0	0
£270 Lancaster Corporation 3 per cent. Stock	227	6	8
£254 2s. 9d. New South Wales 3½ per cent. Inscribed Stock	249	7	5
Balance in the Bank	160	5	2
Guarantee Fund, on Deposit	415	6	7
	£2935	5	10

LIABILITIES.	£	s.	d.
Subscription for 1898 in advance	2	2	0
Balance due to the Treasurer	2	17	2
Abstracts, Oct.–Dec., 1897	29	16	0
Printers	67	14	0
Balance	2832	16	8
	£2935	5	10

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